

XXXI St.Petersburg Summer Meeting in Mathematical Analysis

Abstracts

N. Abuzyarova (Bashkir State University)

Generators for weakly localizable submodules in some weighted modules of entire functions

We consider locally convex spaces of entire functions P that are Fourier-Laplace transform images of the Schwartz space of compactly supported distributions or of some ultradistributions space. These spaces P are topological modules over the ring of polynomials. We establish that each closed submodule J in P is completely defined by its zero set and its indicator segment has at most 2 generators. That is, either J is principal submodule or it coincides with the closure of a set $\{p\varphi + q\psi\}$, where $p, q \in \mathbb{C}[z]$, $\varphi, \psi \in \mathcal{J}$. We also show how to find (construct) the generators φ and ψ .

G. Amosov (Steklov Mathematical Institute)

On covariant operator-valued measures on locally compact groups

Using the Pontryagin duality we construct covariant positive operator-valued measures \mathfrak{M} on the groups $\mathfrak{G} = \hat{G} \times G$ with the projective unitary representation in the Hilbert space $H = L^2(\mu)$, $\mu = \hat{\nu} \times \nu$, where G is a locally compact Abelian group with the Haar measure ν and \hat{G} is its dual with the corresponding Pontryagin Haar measure $\hat{\nu}$. It is shown that given $\rho \in \mathfrak{S}_1(H)$ (the space of nuclear operators) the scalar measure

$$\mu_\rho(B) = \text{Tr}(\rho\mathfrak{M}(B)), \quad B \in \mathfrak{B}(\mathfrak{G})$$

allows to reconstruct ρ .

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S. Astashkin (Samara National Research University)

Sharp distribution estimates for Haar martingale transforms and Rademacher sums and its applications

Let $(\mathcal{F}_n)_{n \geq 0}$ be the standard dyadic filtration on $[0, 1]$. Let $\mathbb{E}_{\mathcal{F}_n}$ be the conditional expectation from $L_1 = L_1[0, 1]$ onto \mathcal{F}_n , $n \geq 0$, and let $\mathbb{E}_{\mathcal{F}_{-1}} = 0$. We present and discuss sharp upper and lower estimates for the distribution function of the martingale transform T defined by

$$Tf = \sum_{m=0}^{\infty} (\mathbb{E}_{\mathcal{F}_{2^m}} f - \mathbb{E}_{\mathcal{F}_{2^{m-1}}} f), \quad f \in L_1,$$

in terms of the classical Calderón operator. In addition, strengthening the well-known Rodin–Semenov theorem, we show that any function from the separable part of the Orlicz space, generated by the function $e^{t^2} - 1$, can be dominated in distribution by a suitable Rademacher sum with coefficients from ℓ_2 . As an application, for a given symmetric function space E on $[0, 1]$, we identify the optimal Banach symmetric range of some classical operators (including the above martingale transform T) acting on E .

F. Avkhadiev (Kazan Federal University)

Geometric problems connected with Hardy-Rellich type inequalities

For domains Ω of the Euclidean space \mathbb{R}^n we consider several quantities $c_p(\Omega) \in [0, \infty)$ defined as sharp constants in Hardy and Rellich type L_p -inequalities on Ω . We will present several known and new results about the following topics of the Geometric Analysis:

- 1) geometric criteria of positivity of quantities $c_p(\Omega) \in [0, \infty)$ in several cases for plane domains;
- 2) a geometric description of families of domains for which one can determine sharp values of the constants $c_p(\Omega)$ for domains $\Omega \subset \mathbb{R}^n$, $n \geq 2$;
- 3) some applications.

Yu. Belov (St.Petersburg State University)

Young type theorem for spaces of analytic functions

In 1981 R. Young proved that the system biorthogonal to a complete and minimal system of exponentials on an interval is always complete. We study this question in different spaces of analytic functions: Fock type spaces, de Branges spaces and Paley-Wiener type spaces with disconnected spectrum. The talk is based on joint works with A. Baranov, A. Borichev and A. Kuznetsov.

E. Dubtsov (St.Petersburg Department of Steklov Mathematical Institute)

A $T(P)$ theorem for Zygmund spaces on domains

Let $D \subset \mathbb{R}^d$ be a bounded Lipschitz domain, ω be a high order modulus of continuity and let T be a convolution Calderón–Zygmund operator. We characterize the bounded restricted operators T_D on the Zygmund space $\mathcal{C}_\omega(D)$. The characterization is based on properties of $T_D P$ for appropriate polynomials P restricted to D . This is joint work with Andrei Vasin.

K. Dyakonov (ICREA and Universitat de Barcelona)

Functions with spectral gaps as extreme or non-extreme points of the unit ball

We discuss the geometry of the unit ball—chiefly the structure of its extreme points—in subspaces of the Hardy spaces H^1 and H^∞ that are formed by functions with prescribed spectral gaps. To be more precise, we fix a subset Λ of $\mathbb{Z}_+ := \{0, 1, 2, \dots\}$ and define $H^1(\Lambda)$ (resp., $H^\infty(\Lambda)$) as the space of functions f in H^1 (resp., H^∞) whose coefficients $\widehat{f}(k)$ vanish whenever $k \notin \Lambda$. Assuming that either Λ or $\mathbb{Z}_+ \setminus \Lambda$ is a finite set, we characterize the extreme (and sometimes exposed) points of the unit ball in $H^1(\Lambda)$ and in $H^\infty(\Lambda)$. Time permitting, some open problems will also be posed.

K. Fedorovskiy (Lomonosov Moscow State University)

B - and C -capacities related with second-order elliptic equations

We plan to consider geometric and metrical properties of B - and C -capacities (that is, capacities defined in classes of bounded and continuous functions) related with homogeneous second-order elliptic equations with constant complex coefficients. These capacities appear quite natural in problems on uniform approximation of functions on compact sets in \mathbb{R}^N , $N \geq 2$, by solutions of the equations under consideration. In the case of harmonic functions (where the operator under consideration is the Laplace operator), the properties of such capacities are well known and they were deeply studied in classical works on potential theory at the first half of XX century. In the general case, these capacities are poorly studied up to now. For a sufficiently wide class of equations under consideration, we plan to present the two-side estimates of B_+ - and C_+ -capacities (i.e., capacities determined by potentials of positive measures) via harmonic capacities in the same dimension. The constructions are based on relatively simple explicit formulae for fundamental solutions of equations under consideration. The talk is based on a joint work in progress with Petr Paramonov (Lomonosov Moscow State University).

D. Gorbachev (Tula State University)

Nikol'skii constants for compact homogeneous spaces

We study the sharp L^p -Nikol'skii constants for the case of Riemannian symmetric manifolds \mathbb{M}^d of rank 1 (see [1]). These spaces are fully classified and include the unit Euclidean sphere \mathbb{S}^d , as well as the projective spaces $\mathbb{P}^d(\mathbb{R})$, $\mathbb{P}^d(\mathbb{C})$, $\mathbb{P}^d(\mathbb{H})$, $\mathbb{P}^{16}(\text{Ca})$. There is a common harmonic analysis on these manifolds, in particular, the subspaces of polynomials $\Pi_n(\mathbb{M}^d)$ of order at most n are defined. In the general case, the sharp L^p -Nikol'skii constant for the subspace $Y \subset L^\infty$ is defined by the equality

$$\mathcal{C}(Y, L^p) = \sup_{f \in (Y \cap L^p) \setminus \{0\}} \frac{\|f\|_\infty}{\|f\|_p}.$$

V.A. Ivanov (1983) gave the asymptotics

$$\mathcal{C}(\Pi_n(\mathbb{M}^d), L^p(\mathbb{M}^d)) \asymp n^{d/p}, \quad n \rightarrow \infty, \quad p \in [1, \infty).$$

For the case of a sphere, this result was significantly improved by the author together with F. Dai and S. Tikhonov (2020):

$$\mathcal{C}(\Pi_n(\mathbb{S}^d), L^p(\mathbb{S}^d)) = \mathcal{C}(\mathcal{E}_1^d, L^p(\mathbb{R}^d))n^{d/p}(1 + o(1)), \quad n \rightarrow \infty, \quad p \in (0, \infty),$$

where \mathcal{E}_1^d is the set of entire functions of exponential spherical type at most 1 bounded on \mathbb{R}^d . M.I. Ganzburg (2020) transferred this equality to the case of the multidimensional torus \mathbb{T}^d and trigonometric polynomials. For $d = 1$, these results follow from the fundamental work of E. Levin and D. Lubinsky (2015).

In a joint work of the author and I.A. Martyanov (2020), the following explicit boundaries of the spherical Nikol'skii constant were proved, which refine the above results for $p \geq 1$:

$$n^{d/p} \leq \frac{\mathcal{C}(\Pi_n(\mathbb{S}^d), L^p(\mathbb{S}^d))}{\mathcal{C}(\mathcal{E}_1^d, L^p(\mathbb{R}^d))} \leq (n + 2\lceil \frac{d+1}{2p} \rceil)^{d/p}, \quad n \in \mathbb{Z}_+, \quad p \in [1, \infty).$$

This result was proved using a one-dimensional version of the problem for the case of a periodic Gegenbauer weight. The development of this method allows us to prove the following general result: for $p \geq 1$

$$n^{d/p} \leq \frac{\mathcal{C}(\Pi_n(\mathbb{M}^d), L^p(\mathbb{M}^d))}{\mathcal{C}(\mathcal{E}_1^d, L^p(\mathbb{R}^d))} \leq (n + \lceil \frac{\alpha_d+3/2}{p} \rceil + \lceil \frac{\beta_d+1/2}{p} \rceil)^{d/p},$$

where $\alpha_d = d/2 - 1$, $\beta_d = d/2 - 1$, $-1/2$, 0 , 1 , 3 for \mathbb{S}^d , $\mathbb{P}^d(\mathbb{R})$, $\mathbb{P}^d(\mathbb{C})$, $\mathbb{P}^d(\mathbb{H})$, $\mathbb{P}^{16}(\text{Ca})$, respectively. The proof of this result is based on the connection of harmonic analysis on \mathbb{M}^d with Jacobi analysis on $[0, \pi]$ and \mathbb{T} with periodic weight $|\sin \frac{t}{2}|^{2\alpha+1} |\cos \frac{t}{2}|^{2\beta+1}$. Also we give related results for the trigonometric Nikol'skii constants in L^p on \mathbb{T} with Jacobi weight and Nikol'skii constants for entire functions of exponential type in L^p on \mathbb{R} with exponential weight.

The work was performed as part of research conducted in the Ural Mathematical Center with the financial support of the Ministry of Science and Higher Education of the Russian Federation (Agreement number 075-02-2021-1383).

[1] Gorbachev D.V. Nikol'skii constants for compact homogeneous spaces, *Chebyshevskii Sbornik* **22** (2021), no. 4, 100–113 (in Russian).

E. Kalita (Institute of Applied Mathematics and Mechanics)

Extension of monotone operators in the nonstandard situation

Let X, Y be separable reflexive Banach spaces, $X \subset Y$ dense. Let $A : X \rightarrow X^*$ be a monotone operator, coercive in the Y norm, that is, $(Au, u)/\|u\|_Y \rightarrow \infty$ as $\|u\|_Y \rightarrow \infty$, $u \in X$. In this settings, an extension of A on Y (generalized pseudomonotone extension of Browder–Hess type) meets such an obstacle as convergence in non-dual spaces ($u_j \rightarrow u$ in Y and $Au_j \rightarrow f$ in X^*). In addition, the natural preliminary domain $D_0 := \{u \in X : Au \in Y^*\}$ of an extension can be a single point $\{0\}$.

We establish the existence of a point-to-point (not multivalued) maximal monotone operator $A_{mm} : Y \rightarrow Y^*$, well-connected with the generalized pseudomonotone extension A_{gpm} , namely, $A_{gpm} A_{mm}^{-1} = Id$ on Y^* .

V. Kapustin (St.Petersburg Department of Steklov Mathematical Institute)

A Hilbert–Polya operator on a Krein space

In our recent paper, we constructed an operator T on a de Branges space, which is a rank-one perturbation of a self-adjoint operator, and whose spectrum coincides with the set of non-trivial zeros of the Riemann zeta function rotated to the real line. A simple fact is that if $XT = AX$, where A is a self-adjoint operator on a Hilbert space and X is bounded and injective, then the point spectrum of T is real. A natural construction related to the de Branges space gives us a self-adjoint operator A on a Krein space, in which the bi-linear form that defines the scalar product is not non-negative. We develop an approach to the theory of the Riemann zeta function based on the construction obtained, in which the key role is played by a special version of the Mellin transform.

I. Kayumov (Kazan Federal University)

Integral means of bounded rational functions defined in domains with rectifiable boundaries

I am going to describe recent results obtained jointly with A. Baranov about the behaviour of integral means of bounded rational functions defined in domains with rectifiable boundaries.

B. Khabibullin (Bashkir State University)

Distribution of zeros for entire functions of exponential type with growth constraints along a closed strip

Let Z be a distribution of points in the complex plane \mathbb{C} , S be a closed strip in \mathbb{C} of finite width, and $M: S \rightarrow [-\infty, +\infty]$ be an extended real function. We investigate the conditions under which there is a non-zero entire function of exponential type f vanishing on Z such that $|f| \leq \exp M$ on S . The following cases will be considered.

- Beurling–Malliavin case: S is the real axis and $\int_{-\infty}^{+\infty} \frac{|M(x)|}{1+x^2} dx < +\infty$.
- Rubel–Malliavin case: S is a straight line and M is the restriction to S of a function $\ln |g|$, where g is the entire function of exponential type.
- Subharmonic case: M is the restriction to S of subharmonic function M on \mathbb{C} such that $M(z) \leq O(|z|)$ as $z \rightarrow \infty$.

We also plan to discuss possible ways of passing to arbitrary functions M on the strip S and applying these results to questions of approximation.

E. Korotyaev (St.Petersburg State University)

Schrödinger operators periodic in octants

We consider Schrödinger operators with periodic potentials in the positive quadrant on the plane with Dirichlet boundary conditions. We show that, for any integer N and any interval I , there exists a periodic potential such that the Schrödinger operator has N eigenvalues counted with multiplicity in this interval and there is no other spectrum in the interval. Furthermore, to the right and to the left of it there is an essential spectrum. Moreover, we prove similar results for Schrödinger operators for a product of an orthant and Euclidean space. The proof is based on the inverse spectral theory for Hill operators on the real line.

This first part is based on the joint paper [1] with Jacob Schach Moller (Danmark). Secondly, we consider similar problems for discrete case, published in [2].

[1] Korotyaev E., Moller J.S. Schrödinger operators periodic in octants, Letters in Mathematical Physics **111** (2021), no. 2, 55.

[2] Korotyaev E. Eigenvalues of periodic difference operators on lattice octants, Journal of Mathematical Analysis and Applications **500** (2021), no. 2, 125138.

A. Kuznetsova (Kazan Federal University)

On endomorphisms of the Toeplitz algebra

We consider the semigroup $\text{End}_0 \mathcal{T}$ of the faithful endomorphisms of the Toeplitz algebra. We determine the connection between $\text{End}_0 \mathcal{T}$ and the semigroup of nonunitary isometries of the Toeplitz algebra. We introduce the notion of the index of an endomorphism, which gives the grading on $\text{End}_0 \mathcal{T}$, and we prove that the surjective semigroup homomorphism exists between $\text{End}_0 \mathcal{T}$ and the subsemigroup of the semigroup of endomorphisms of the algebra of all continuous functions on the unit circle.

Using a finite Blaschke product, one can set the endomorphism on \mathcal{T} , and all these endomorphisms form the subsemigroup $\text{End}_{\mathcal{B}} \mathcal{T} \subset \text{End}_0 \mathcal{T}$. We describe the subalgebra of \mathcal{T} which is invariant under $\text{End}_{\mathcal{B}} \mathcal{T}$. Joint work with Tamara Grigoryan (Kazan State Power Engineering University).

V. Lebedev (National Research University Higher School of Economics)

Changes of variable and Fourier Multipliers

We consider the algebras M_p of L_p -Fourier multipliers and show that, for every bounded continuous function f on \mathbb{R}^d , there exists a self-homeomorphism h of \mathbb{R}^d such that the superposition $f \circ h$ is in $M_p(\mathbb{R}^d)$ for all p , $1 < p < \infty$. Moreover, under certain assumptions on a family K of continuous functions, one h will suffice for all $f \in K$. A similar result holds for the functions on the torus \mathbb{T}^d . In the one-dimensional case,

the result is sharp in the sense that it does not extend to the case of $p = 1$ (this follows from the known solution of Luzin's problem related to the Wiener algebra). We shall briefly outline the idea of the proof of the result and discuss some of its consequences and related open questions. Joint work with Alexander Olevskii.

M. Malamud (Peoples Friendship University of Russia)

To the Birman problem of positive symmetric operators

Let $A \geq 0$ be a closed non-negative densely defined symmetric operator in a Hilbert space \mathfrak{H} which is assumed to be positive definite, $(Af, f) \geq m_A \|f\|^2$. According to Krein the set of all non-negative selfadjoint extensions contains the maximal (the Friedrichs) and the minimal (the Krein) extensions \widehat{A}_F and \widehat{A}_K . The Friedrichs extension \widehat{A}_F preserves the lower bound $m_{\widehat{A}_F} = m_A$, while the Krein extension admits a representation $\widehat{A}_K = \widehat{A}'_K \oplus (\mathbb{O} \upharpoonright \ker A^*)$. The operator \widehat{A}'_K is called the reduced Krein extension.

Birman posed the following problem: Is it true that the compactness of the inverse operator A^{-1} implies the compactness of the inverse of the Friedrichs extension $(\widehat{A}_F)^{-1}$?

A negative solution to this problem will be discussed. We show that the compactness of A^{-1} ensures the discreteness of the spectrum of the reduced Krein extension. Moreover, we present explicit examples of positive elliptic differential operators demonstrating this effect.

P. Mozolyako (St.Petersburg State University)

Weighted Hardy operator on the poly-tree

Let Γ be a poly-tree, i.e., a collection of dyadic rectangles on \mathbb{R}^n (Cartesian product of usual dyadic intervals on \mathbb{R}) with natural order by inclusion. The Hardy operator and its "adjoint" are

$$\mathbf{I}f(R) := \sum_{R \subset Q} f(Q), \quad \mathbf{I}^*f(Q) := \sum_{R \subset Q} f(R).$$

We are investigating the action of this operator from $L^2(\Gamma, w^{-1})$ to $L^2(\Gamma, \mu)$, or, which is the same, \mathbf{I}^* from $L^2(\Gamma, \mu^{-1})$ to $L^2(\Gamma, w)$, where w and μ are just collections of non-negative weights attached to the elements of Γ . If for given μ, w the Hardy operator is bounded, we call (μ, w) *the trace measure-weight pair*.

Our main goal is to provide a characterization of such pairs. They appear naturally in a number of settings—Hardy operators on $[0, +\infty)^n$, Carleson measures for analytic and harmonic weighted Dirichlet spaces on the polydisc, maximal operators on dyadic rectangles, etc.

We plan to give a short review of known results and discuss some recent developments regarding the case of non-product weights.

I. Musin (Institute of Mathematics with Computing Centre, Ufa)

On a Denjoy–Carleman class of periodic ultradifferentiable functions

A space $J(\mathcal{H})$ of 2π -periodic infinitely differentiable functions on the real line with given estimates on their derivatives defined with a help of a family \mathcal{H} of convex nondecreasing functions on $[0, \infty)$ will be considered in the talk. A description of $J(\mathcal{H})$ in terms of the best trigonometric approximations and the rate of decrease of the Fourier coefficients will be given. Examples of a family of convex functions \mathcal{H} will be presented. Also some other problems motivated by the researches of P.L. Ul'yanov [1] will be considered.

[1] Ul'yanov P.L. On classes of infinitely differentiable functions, Sb. Math. **70** (1991), no. 1, 11–30.

S. Nasyrov (Kazan Federal University)

Comparison of intrinsic metrics in convex polygonal planar domains

A local study leads to investigation of the relationship between the conformal radius at an arbitrary point of a planar domain and the distance of the point to the boundary. We study the ratio of these characteristics and describe the subset of a given polygonal domains where the ratio attains its maximal value. This gives

us an information on the sharp constants in the inequalities involving the triangular ratio metric and the hyperbolic metric. The talk is based on results of joint work with D.Dautova, R.Kargar, and M.Vuorinen.

V. Peller (St.Petersburg State University)

Triangular projection on Schatten–von Neumann classes S_p

I am going to speak about my recent joint results with A.B. Aleksandrov. The main subject of the talk is the study of properties of the triangular projection on Schatten–von Neumann classes S_p with $p < 1$. In particular, we solve a problem that was posed recently by B.S. Kashin.

A. Posadsky (Moscow Institute of Physics and Technology)

Conformal mapping onto a polygon with several cuts

In the geometric function theory, one of important problems is finding accessory parameters in the Schwarz–Christoffel integrals which map conformally a canonic domain onto a given polygon. P.P.Kufarev ([1], see also [2]) suggested an approximate method based on the Löwner parametric method. A modification of Kufarev’s method was proposed in [3] where the authors proposed to cut a polygon from another one.

Here we present a modification of Gutlyanskii and Zaidan’s method; to obtain a given polygon we use cutting along several rectilinear cuts. We derive the corresponding Löwner differential equation [2]:

$$\frac{\partial f(z, t)}{\partial t} = -\frac{\partial f(z, t)}{\partial z} z(z-1) \sum_{i=1}^m C_i(t) \frac{\lambda_i(t) - 1}{\lambda_i(t) - z}$$

and, with its help, deduce a system of ODE for accessory parameters. It is shown that if the side lengths depend smoothly on each other at fixed angles, then the family of mappings is smooth with respect to the to changing length.

The advantage of the suggested method is that in the Kufarev’s method we have to solve a number of systems of ODEs, and in our method the number of steps can be reduced to two. The system of ODEs is more complicated but will not essentially differ from case of one cut.

We give some numerical examples showing the efficiency of our method. Joint work with S.R. Nasyrov (Kazan Federal University).

[1] Kufarev P.P. A method for determining the parameters of the Schwarz–Christoffel integral, Dokl. Akad. Nauk SSSR **57** (1947), 535–537 (in Russian).

[2] Aleksandrov I.A. Parametrical Extensions in the Theory of Univalent Functions, Moscow, Nauka, 1976 (in Russian).

[3] Gutlyanskii V.Ya., Zaidan A.O. On conformal mapping of polygonal regions, Ukr. Math. J. **45** (1993), no. 11, 1669–1680.

N. Shirokov (St.Petersburg State University)

Conformal mappings of areas which are geometrically close to a disk

We consider an area that differs from the unit disk by a finite collection of continua with small diameters, and the conformal mapping of the area onto the unit disk, which takes 0 to 0 and -1 to -1. We prove that on the unit circle, the mapping differs from the identity mapping by a small quantity that depends on the continua. This is joint work with M.S. Kuznetsova.

D. Stolyarov (St.Petersburg State University)

Trace inequalities for functions and martingales

One may define the trace of a Sobolev function in the space $W_1^1(\mathbb{R}^d)$ on an arbitrary set of finite $(d-1)$ -dimensional Hausdorff measure. The inequality that expresses this principle was proved by Maz’ja and (independently and slightly later) by Meyers and Ziemer in mid 70s. The inequality itself allows (employing other considerations as well) to say much more about the structure of functions in the classes W_1^1 and BV. Let now a function or a vector field satisfy a more involved differential condition (say, a field is a normal charge, which means its divergence is a finite signed measure). On which sets are we able to define the

trace of such an object, and how does the corresponding inequality look like? I will not provide full or even satisfactory answer to this question. However, I will show a discrete model of the problem, in which the things become more transparent, consider an interesting particular case, and formulate several conjectures.

S. Strakhov (Samara University)

On two types of symmetric spaces

Let X be a symmetric space (for example, the Lebesgue L_p , Lorentz $\Lambda_p(\varphi)$ and Orlicz L_F spaces are symmetric) on $[0, 1]$ and let $H \subset X$ be its closed subspace. In our work, we study the following numerical characteristic of the subspace:

$$\eta_X(H) = \lim_{\tau \rightarrow 0} \sup_{x \in H, x \neq 0} \sup_{e \subset [0,1], \mu(e) \leq \tau} \frac{\|x\chi_e\|}{\|x\|}. \quad (1)$$

It was shown in [1] that L_p and $\Lambda_p(\varphi)$ for $1 \leq p < 2$ are binary, i.e., $\eta_X(H)$ for them takes only two values—0 and 1. In the case where $2 \leq p < \infty$, the introduced characteristic takes infinitely many values in these spaces. This follows from the sufficient conditions for the nonbinarity of X obtained in the same work.

More general sufficient conditions for nonbinarity of a symmetric space will be given in the talk. We will also show the connection between the introduced characteristic and the metric of a spherical opening on the space of all subspaces of a symmetric space.

[1] Novikov S. Ya., Semenov E. M., Tokarev E. V. The structure of subspaces of the space $\Lambda_p(\varphi)$, Dokl. Akad. Nauk SSSR **247** (1979), 552–554; English transl.: Soviet Math. Dokl. **20** (1979), 760–761.

E. Turilova (Kazan Federal University)

Spectral order in operator algebras theory

The talk summarizes recent results on the spectral order on AW^* -algebras and von Neumann algebras with the following aims:

- to present a “non-orthodox” order on operators that can replace the standard order and has an interesting physical content;
- to show that the most natural context for studying the spectral order is given by abstract AW^* -algebras;
- to demonstrate that the spectral order organizes operator effect algebra into a complete lattice that naturally contains projection lattice as a sublattice;
- to present deep results on symmetries of the spectral order that can be viewed as “unsharp” generalizations of famous Wigner’s and Dye’s Theorems on symmetries of a quantum system.

We initiate also study of spectral order on Jordan triplets. The order given on the tripotents is extended to the spectral order on the triplets. We show that Jordan triplets equipped with a spectral order are not a lattice, but preserve the Olson “moment” characteristic.

A. Ulitskaya (St.Petersburg State University)

Sharp inequalities for mean square approximation of classes of convolutions by spaces of shifts

We obtain sharp inequalities for L_2 -approximation of convolution classes by spaces generated by equidistant shifts of a single function. Moreover, we give a complete description of spaces of shifts providing these estimates. Necessary and sufficient conditions of extremality are formulated in terms of Fourier coefficients (in periodic space L_2) or Fourier transform (in $L_2(\mathbb{R})$) of convolution kernel and function generating the space of shifts. In addition, we indicate easily verifiable conditions that are sufficient for the fulfillment of the inequalities under consideration and give examples of kernels and extremal subspaces satisfying these conditions.

I. Vasilyev (Université Paris Saclay and PDMI RAS)

On one strengthening of the First Beurling–Malliavin Theorem

I will discuss a new sufficient condition for a function to belong to the class of so-called Beurling–Malliavin majorants. This condition consists of the (classical) convergence of the logarithmic integral of the function and of the (new) anisotropic weighted Sobolev type regularity, imposed on its logarithm.

O. Vinogradov (St.Petersburg State University)

Sharp Bernstein type inequalities and interpolation formulas

The direct method to prove the classical sharp Bernstein inequality is the M.Riesz interpolation formula, which represents the derivative of a trigonometric polynomial as a linear combination of its equidistant shifts. We discuss the construction of such interpolation formulas and sharp Bernstein type inequalities in the setting of Dunkl and Jacobi–Dunkl transform and in the spline setting.

R. Yulmukhametov (Institute of Mathematics with Computing Centre, Ufa)

On a criterion for the existence of unconditional bases of reproducing kernels in Fock spaces with radial regular weight

We consider the spaces \mathcal{F}_φ of entire functions f such that $fe^{-\varphi} \in L_2(\mathbb{C})$, where $\varphi(z) = \varphi(|z|)$ is a radial subharmonic function with some regularity property. We prove that \mathcal{F}_φ has Riesz basis of normalized reproducing kernels if and only if $(\varphi(e^r))''$ is bounded above.

A. Zhang (University of Wisconsin-Madison)

Complex analytic approach to spectral problems for differential operators

This talk will be about applications of complex function theory to inverse spectral problems for canonical systems, which constitute a broad class of second order differential equations. I will start with the basics of Krein–de Branges theory, then present an algorithm developed by Makarov and Poltoratski for locally-finite periodic spectral measures. Finally, I will extend the algorithm to certain classes of non-periodic spectral measures and present several examples. This is joint work with Alexei Poltoratski.