XXIX St. Petersburg Summer Meeting in Mathematical Analysis

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ABSTRACTS

Nataliya Abuzyarova (*Bashkir State University, Russia*)

On the perturbations of zero sets preserving Fourier–Laplace transform classes of entire functions

We consider four different subsets of the Schwartz algebra of entire functions defined by some restrictions on the zero sets or (and) some growth conditions. We establish that bounded perturbations of the real parts and logarithmically increasing ones of the imaginary parts preserve function in each of the considered classes. As an application, we show that such perturbations of the synthesable complex sequence preserve this property.

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Alexander Bufetov (*Steklov Institute, Russia and Aix-Marseille University, France*)

The sine-process has excess one

The main result of the talk is that almost every realization of the sine-process with one particle removed is a uniqueness set for the Paley–Wiener space; with two particles removed, a zero set for the Paley–Wiener space. The talk is based on the preprint https://arxiv.org/abs/1912.13454.

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Konstantin Dyakonov (*ICREA and Universitat de Barcelona*)

Lacunary polynomials in L^1: geometry of the unit sphere

Let Λ be a finite set of nonnegative integers, and let $\mathcal{P}(\Lambda)$ be the linear hull of the monomials z^k with $k \in \Lambda$, viewed as a subspace of L^1 on the unit circle. We characterize the extreme and exposed points of the unit ball in $\mathcal{P}(\Lambda)$.

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Jean Esterle (*University of Bordeaux, France*)

Do some nontrivial closed invariant subspaces have the division property?

We consider Banach spaces E of functions holomorphic on the open unit disc $\mathbb{D}$ such that the unilateral shift S and the backward shift T are bounded

on E. Assuming that the spectra of S and T are equal to the closed unit disc we discuss the existence of closed z-invariant subspaces N of E having the "division property", which means that the function $z \mapsto f(z)/(z-a)$ belongs to N for every $f \in N$ vanishing at a. This question is related to the existence of nontrivial bi-invariant subspaces of Banach spaces of hyperfunctions on the unit circle $\mathbb{T}$.

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Evgenii Dubtsov (*St. Petersburg Department of Steklov Mathematical Institute*)

Comparison of Clark measures in several variables

Let $\mathbb{D}$ denote the unit disc of $\mathbb{C}$ and let Ω denote the unit ball B_n of $\mathbb{C}^n$ or the unit polydisc $\mathbb{D}^n$, $n \geq 2$. Given a non-constant holomorphic function $b : \Omega \to \mathbb{D}$, we study the corresponding family $\sigma_\alpha[b]$, $\alpha \in \partial\mathbb{D}$, of Clark measures on $\partial\Omega$. For $\Omega = B_n$ and an inner function $I : B_n \to \mathbb{D}$, we show that the property $\sigma_1[I] \ll \sigma_1[b]$ is related to the membership of an appropriate function in the de Branges–Rovnyak space $\mathcal{H}(b)$.

This is a joint work with A.B. Aleksandrov.

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Konstantin Fedorovskiy (*Bauman Moscow State Technical University and Saint Petersburg State University, Russia*)

Chui's conjecture in Bergman spaces

In the talk we present a solution of Chui's problem on the simplest fractions (i.e., sums of Cauchy kernels with unit coefficients) in weighted (Hilbert) Bergman spaces. Namely, for a wide class of weights, it will be shown that for every N, the simplest fractions with N poles on the unit circle have minimal norm if and only if the poles are equidistributed on the circle. We present sharp asymptotics of these norms. Finally, we describe the closure of the simplest fractions in weighted Bergman spaces. The talk is based on a recent joint work with E. Abakumov and A. Borichev.

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Kunyu Guo (*Fudan University, China*)

The Beurling–Wintner problem and analytic number theory

In this talk, I will introduce an analytic number theoretic approach toward a classical analysis problem raised by Beurling and Wintner independently in the 1940s. This problem is to seek $\varphi \in L^2(0,1)$ for which the system $\{\varphi(kx) : k = 1, 2, \cdots\}$ is complete in $L^2(0,1)$, where φ is identified with its odd 2-periodic extension on $\mathbb{R}$. We completely solve the Beurling–Wintner problem for step functions with rational jump discontinuities. As a byproduct, we completely

solve the rational number version of the Kolzov completeness problem. We also exhibit some examples and applications. This is a joint work with H. Dan.

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Michael Hartz (*University of Saarland, Germany*)

The column-row property for complete Pick spaces

Complete Pick spaces form a class of Hilbert function spaces including the Hardy space on the disc, the classical Dirichlet space and also the Drury-Arveson space on the ball. They were first studied because of their connection to interpolation problems. More recently, it turned out that some apparently function theoretic properties of the Hardy space in fact extend to more general complete Pick spaces. In a number of cases, these results required an additional hypothesis, called the column-row property.

I will talk about a result showing that the column-row property is automatic for complete Pick spaces. Moreover, I will sketch applications to weak products, interpolating sequences and de Branges–Rovnyak spaces on the ball.

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Ilgiz Kayumov (*Kazan Federal University, Russia*)

Bohr type inequalities and their generalizations

I am going to describe results concerning the classcical Bohr inequality and its generalizations including results for subordinated and quasi-subordinated functions in the unit disk. A couple of conjectures concerning weighted Bohr sums will be formulated.

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Bulat Khabibullin (*Bashkir State University, Russia*)

Completeness of exponential systems in spaces of holomorphic functions on convex domains

Let $O \neq \varnothing$ be an open set in the complex plane $\mathbb{C}$, and $\mathsf{Z} := (\mathsf{z}_j)_{j=1,2,\dots}$ be a sequence of pairwise distinct points in $\mathbb{C}$ without limit points in $\mathbb{C}$. The exponential system $\operatorname{Exp}^{\mathsf{Z}} := \left\{ z \underset{z\,\in\,\mathbb{C}}{\longmapsto} e^{\mathsf{z}_j z} \right\}_{j=1,2,\dots}$ is *complete* in the space $\operatorname{Hol}(O)$ of holomorphic functions on O equipped with the topology of uniform convergence on compact subsets of O if the closure of the linear span of $\operatorname{Exp}^{\mathsf{Z}}$ coincides with $\operatorname{Hol}(O)$. For *unbounded convex* O and $\mathsf{Z} \subset \mathbb{C}$, our completeness criterion of $\operatorname{Exp}^{\mathsf{Z}}$ in $\operatorname{Hol}(O)$ was obtained in the late 1980s exclusively in terms of the upper density and the logarithmic block density of Z on the one side and the width of O in direction on the other. Our 2020 criterion is new for *any bounded convex* $O \subset \mathbb{C}$, be it the unit disk, a polygon, etc. Below $\mathbb{R}$ is the real axis, $\mathbb{R}^+ := \{r \in \mathbb{R} \colon r \geq 0\}$, and $\mathsf{s}_S(t) := \underset{t\,\in\,\mathbb{R}}{\sup_{z\in S}} \operatorname{Re}(ze^{-it})$ is the support function of $S \subset \mathbb{C}$.

Completeness Criterion 2020. *Let O be bounded and convex. The system* $\mathrm{Exp}^{\mathsf{Z}}$ *is complete in* $\mathrm{Hol}(O)$ *if and only if there exist numbers $p \in [0,1)$ and $r_0 > 0$, a integrable non-zero function $g\colon \mathbb{R}^+ \setminus 0 \to \mathbb{R}^+$ of class C^2 satisfying $g(r) \underset{r \geq r_0}{\equiv} 0$, $\lim\limits_{r\to 0+} \dfrac{g(r)}{|\ln r|} < +\infty$, $r^2 g''(r) + r g'(r) \underset{r \in \mathbb{R}^+ \setminus 0}{\geq} p^2 g(r)$, and also a bounded convex open subset $S \subset \mathbb{C}$ with $0 \in S$ and with the boundary ∂S of class C^2 satisfying $\mathsf{s}''_S(t) \underset{t \in \mathbb{R}}{\geq} -p^2 \mathsf{s}_S(t)$ such that*

$$\limsup_{R\to+\infty} \frac{\pi}{R} \sum_j g\Big(\frac{|\mathsf{z}_j|}{R}\Big) \mathsf{s}_S(\arg \mathsf{z}_j) \geq \int\limits_0^{+\infty} g(r)\,\mathrm{d}r \left(\frac{1}{2} \int\limits_0^{2\pi} \mathsf{s}_O(t) \big(\mathsf{s}_S(t) + \mathsf{s}''_S(t)\big)\,\mathrm{d}t \right)$$

where the last bracket is the mixed area of the closures of O and S, and $\mathsf{s}_S(t) + \mathsf{s}''_S(t)$ is the radius of curvature of ∂S at the point of tangency of the support line to ∂S, orthogonal to the radius-vector of the point e^{it}.

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Diana Khammatova (*Kazan Federal University, Russia*)

The Bohr inequality for the generalized Cesáro averaging operators

The main aim of our work is to prove a generalization of the classical Bohr theorem. As an application, we obtain a counterpart of Bohr theorem for the generalized Cesáro operator.

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Evgeny Korotyaev (*Saint Petersburg State University, Russia*)

Inverse scattering for Schrödinger operators with compactly supported potentials on half line

We solve inverse scattering problem for Schrödinger operators with compactly supported potentials on the half line. We discretize S-matrix: we take the value of the S-matrix on some infinite sequence of positive real numbers. Using this sequence obtained from S-matrix we recover uniquely the potential by a new explict formula, without the Gelfand–Levitan–Marchenko equation.

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Andrei Lishanskii (*Saint Petersburg State University, Russia*)

Hypercyclicity of Toeplitz operators

We study hypercyclicity of Toeplitz operators acting on the Hardy space $H^2(\mathbb{D})$ with symbols of the form $\Phi(z) = R(\frac{1}{z}) + \varphi(z)$, where R is a rational function. In 2016 A. Baranov and A. Lishanskii found necessary and also sufficient conditions for hypercyclicity of Toeplitz operators with polynomial antianalytic part. In this talk we discuss new sufficient hypercyclicity conditions for the operators with rational antianalytic part using some results of

B. Solomyak. The talk is based on a joint work with E. Abakumov, A. Baranov and S. Charpentier.

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Lev Maergoiz (*Krasnoyarsk Science Center of the Siberian Branch of the Russian Academy of Sciences*)

The relationship between the indicator of an entire function of finite order and its Taylor coefficients

Let $f(z) = \sum_{k=0}^{\infty} c_k z^k/\Gamma[(k+1)/\rho],\ z \in \mathbb{C}$, be an entire function of any finite order $\rho > 0$ and normal type σ. There are well known formulas expressing ρ and σ in terms of the rate of decrease of the Taylor coefficients of f. The talk discusses the same problem for the indicator of f. We associate with f the Puiseux series $F(p) = \sum_{k=0}^{\infty} c_k/p^{(k+1)/\rho}$. We present a solution based on the multivalent variant of G. Pólya's theorem describing the relation between the indicator and the conjugate diagrams for entire functions of exponential type (see [1]). This analogue of Pólya's theorem makes possible to solve the problem using properties of the domain of analytic continuation of the Puiseux series F. This is a developement of investigatons of [2] where the author associated with f the Laurent series.

[1] L. S. Maergoiz, Analytic continuation methods for multivalued functions of one variable and their application to the solution of algebraic equations, *Proc. Steklov Inst. Math.* **308**, Suppl. 1 (2020), 135–151.

[2] G. G. Braichev, Calculation of the indicator of an entire function of fractional order in terms of its Taylor coefficients, *Ukrain. Matem. J.*, **45:6** (1993), 854–858 (in Russian).

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Andrei Martinez-Finkelstein (*Baylor University, USA*)

Curves inscribed in polygons: beyond Poncelet's theorem

Poncelet's Theorem is one of the most beautiful and well known results from projective geometry. In the last few decades, the relationship between Poncelet's Theorem and other mathematical objects, such as Blaschke products or numerical range of completely non-unitary contractions, has been the focus of extensive research. Recently, another connection, now with the theory of orthogonal polynomials on the unit circle has been revealed. These interconnections allow us to prove several new results, to interpret the existing theory in a new context, and also to understand further connections with other areas of geometry and analysis.

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Nikolai Osipov (*St. Petersburg Department of Steklov Mathematical Institute*)

Bellman function for Gundy theorem

We show that Burkholder's approach can be extended to unitary operators satisfying Gundy conditions (that is a martingale analogue of Calderón–Zygmund conditions).

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Vladimir Peller (*Saint Petersburg State University, Russia*)

Lipschitz and Hölder type estimates for functions of commuting dissipative operators

I am going to speak about joint results with A.B. Aleksandrov on estimates of functions of commuting dissipative operators under perturbation.

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Sergey Platonov (*Petrozavodsk State University*)

Some properties of the Fourier transform of Dini–Lipschitz functions on locally compact Vilenkin groups

Let G be a locally compact bounded Vilenkin group, Γ be the dual group of G. Suppose that a function $f(x)$ belongs to the the Lebesgue class $L^p(G)$, $1 < p \leq 2$, and let $\widehat{f}(\xi)$ be the Fourier transform of f. In this talk we discuss an answer to the following problem: if the function f belongs to the Dini–Lipschitz class $DLip(\alpha, \beta, p; G)$, $\alpha > 0$, $\beta \in \mathbb{R}$, then for which values of r can we guarantee that $\widehat{f} \in L^r(\Gamma)$? The result is an analogue of one classical theorem of E. Titchmarsh about the Fourier transform of functions from the Lipschitz classes on $\mathbb{R}$.

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Alexander Pushnitski (*King's College, United Kingdom*)

Kato smoothness and functions of perturbed self-adjoint operators

I will discuss a new class of estimates for the operator norm and the Schatten norm of $f(A) - f(B)$, where A and B are self-adjoint operators on a Hilbert space. These estimates utilise ideas of scattering theory and involve conditions on A and B in terms of the Kato smoothness. They allow for a much wider class of functions f (including some unbounded ones) than the previously available results. This is joint work with Rupert Frank (LMU Munich and Caltech).

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Oleg Reinov (*Saint Petersburg State University, Russia*)

A Banach lattice with the approximation property that does not have the bounded approximation property

For the first time an example of a Banach space with the approximation property but without the bounded approximation property was obtained by Figiel and Johnson in 1973. We present the first example of a Banach lattice with the approximation property that does not have the bounded approximation property. As a consequence, one gets the existence of an integral operator (in the sense of Grothendieck) in a Banach lattice which is not strictly integral (the existence of such an operator in a Banach space follows from the construction of Figiel and Johnson).

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Eugene Shargorodsky (*King's College, United Kingdom*)

Essential norms of Toeplitz operators and related estimates

It is well known that the essential norm of a Toeplitz operator on the Hardy space $H^p(\mathbb{T})$, $1 < p < \infty$, is greater than or equal to the supremum norm of its symbol. In 1988, A. Böttcher, N. Krupnik, and B. Silbermann posed a question on whether or not the equality holds in the case of continuous symbols. We answer this question in the negative. On the other hand, we show that the essential norm of a Toeplitz operator with a continuous symbol is less than or equal to twice the supremum norm of the symbol and prove more precise p-dependent estimates. We also discuss some related estimates for compact operators on L^p spaces obtained jointly with T. Sharia and some open questions.

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Nikolai Shirokov (*Saint Petersburg State University, Russia*)

Nevanlinna factorization in weighted spaces of analytic functions of variable smoothness

Let f be a function analytic in the unit disc and continuous in its closure, and let $f = IF$ be its Nevanlinna factorization, where I is the inner factor and F is an outer function. In the talk we discuss the relation between the factor I and the decay of the function F near the spectrum of I and the description of outer functions F in the case when f belongs to a space of functions, defined in terms of some integral condition with variable exponent.

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Mikhail Sodin (*Tel Aviv University, Israel*)

Fourier uniqueness and non-uniqueness pairs

Motivated by a remarkable discovery by Radchenko and Viazovska and by a recent work by Ramos and Sousa, we find conditions sufficient for a pair of

discrete subsets of the real axis to be a uniqueness or a non-uniqueness pair for the Fourier transform. These conditions are not too far from each other. The uniqueness theorem can be upgraded to the frame bound and an interpolation formula, which in turn produce an abundance of Poisson-like formulas.

This is a report on a joint work in progress with Aleksei Kulikov and Fedor Nazarov.

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Sergei Treil (*Brown University, USA*)

Matrix Clark measures, singular spectrum and matrix Carathéodory angular derivatives

Matrix Clark measures appear naturally in the study of finite rank perturbations, as well as in the theory of de Branges–Rovnyak spaces. Both approaches give the same measure if the characteristic function θ satisfies $\theta(0) = 0$; in the general case the measures differ by constant normalizing factors.

In the talk I describe recent results about fine properties of the matrix Clark measures, especially of the singular parts of such measures. The notion of the Carathéodory angular derivative will be introduced for the matrix Clark measures, and connection with point masses will be discussed.

While the results generalize known statements about scalar Clark measures, the generalization is far from trivial: the ideas of *directionality* play an important role and present a major obstacle in the matrix case.

The talk is based on a joint work with C. Liaw and R.T.W. Martin.

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