Euler International Mathematical Institute

# XXVIII St. Petersburg Summer Meeting in Mathematical Analysis

**June 25-30, 2019**

The conference is supported by Simons Foundation and by “Native towns”, a social investment program of PJSC “Gazprom Neft”.

## CONFERENCE PROGRAM

### TUESDAY, 25

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|---|---|
| 10:00–10:55 | REGISTRATION |
| 11:00–11:45 | **N. Nikolski.** *Quantitative constraints for signatures of unconditional bases and frames.* |
| 12:10–12:55 | **V. Eiderman.** *A "rare" plane set with Hausdorff dimension* 2*.* |

### Lunch

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|---|---|
| 15:00–15:45 | **S. Treil.** *Convex body domination and weighted estimates with matrix-valued weights.* |
| 16:00–16:45 | **O. Ivrii.** *Homogenization of random quasiconformal mappings and random Delauney triangulations.* |
| 17:00–17:25 | **A. Mirotin.** *On spectral representation for multidimensional normal Hausdorff operators.* |
| 17:30–17:55 | **A. Kuznetsova.** *Local groups and related $C^*$-algebras.* |
| 18:00 | WELCOME PARTY |

## WEDNESDAY, 26

11:00–11:45 **E. Korotyaev.** *Inverse resonance scattering for the Laplacian on rotationally symmetric manifolds.*

12:10–12:55 **B. Khabibullin.** *Affine balayage and zeros of holomorphic functions.*

### Lunch

15:00–15:45 **V. Ivrii.** *Complete spectral asymptotics for periodic and almost periodic perturbations of constant operators and Bethe–Sommerfeld conjecture in semiclassical settings.*

16:00–16:45 **S. Platonov.** *Fourier–Jacobi harmonic analysis and some problems of approximation of funcions on the half-axis in $L_2$ metric.*

17:00–17:25 **R. Akgün.** *On Jackson inequalities in Lebesgue spaces with Muckenhoupt weights.*

17:30–17:55 **E. Kalita.** *Estimates of some selfadjoint operators in the neighborhood of $L_2$.*

## THURSDAY, 27

11:00–11:45 **K. Fedorovskiy.** *Approximation by simplest fractions and Chui conjecture.*

12:10–12:55 **A. Baranov.** *Spectral theory of rank one perturbations of normal operators.*

## Lunch

15:00 BOAT TRIP

## FRIDAY, 28

11:00–11:45 **E. Doubtsov.** *Clark measures in several variables.*

12:10–12:55 **D. Stolyarov.** *Martingale models for cancelling and weakly cancelling differential operators.*

## Lunch

15:00–15:45 **M. Balashov.** *Proximal smoothness of the real Stiefel manifold.*

16:00–16:45 **O. Vinogradov.** *Two-sided estimates for deviations of quasiprojectors.*

17:00–17:25 **I. Vasilyev.** *On the multidimensional Havin–Shamoyan–Carleson–Jacobs theorem.*

17:30–17:55 **L. Maergoiz.** *Analytic continuation methods for multivalued functions of one variable and their application to the solution of algebraic equations.*

## SATURDAY, 29

11:00–11:45 **P. Ohrysko.** *Inversion problem in measure algebras.*

12:10–12:55 **L. Slavin.** *The Helson–Szego theorem and the* $A_2$ *condition.*

### Lunch

15:00–15:45 **G. Amosov.** *On singular perturbation of the semigroup of shifts on the algebra of canonical anticommutation relations.*

15:55–16:20 **A. Mokeev.** *Non-commutative operator graphs generated by orbits of unitary groups.*

16:25–16:50 **S. Simonov.** *Wave models of metric spaces.*

17:05–17:30 **T. Andreeva.** *On the surjectivity of convolution operators on holomorphic weighted spaces in bounded convex domains.*

17:35–18:00 **V. Kapustin.** *Hitt's theorem for the half-plane and rational interpolation.*

19:00 CONFERENCE DINNER

## SUNDAY, 30

11:00–11:45 **V. Peller.** *Functions of pairs of contractions under perturbations.*

12:10–12:55 **H. Hedenmalm.** *Planar orthogonal polynomials, random normal matrices and arithmetic jellium.*

# ABSTRACTS

**R. Akgün.** *On Jackson inequalities in Lebesgue spaces with Muckenhoupt weights.*

Jackson inequalities are established for functions of Lebesgue spaces with Muckenhoupt weights. Simultaneous approximation by polynomials is considered. Some uniform norm inequalities are transferred to weighted Lebesgue spaces.

**G. Amosov.** *On singular perturbation of the semigroup of shifts on the algebra of canonical anticommutation relations.* (Joint work with E.O. Kholmogorov.)

Let $\mathfrak{A}(H)$ denote the algebra of canonical anticommutation relations over the Hilbert space $H = L^2(\mathbb{R}_+)$ generated by the creation and annihilation operators $a^*(f)$, $a(g)$ satisfying the relations

$$a^*(f)a(g) + a(g)a^*(f) = (g, f)I,$$

$$a(f)a(g) + a(g)a(f) = a^*(f)a^*(g) + a^*(g)a^*(f) = 0,$$

$f, g \in H$. The standard representation of $\mathfrak{A}(H)$ can be realised in the antisymmetric Fock space

$$F(H) = \{\mathbb{C}\Omega\} \oplus H \oplus \cdots \oplus H^{\otimes_a^n} \oplus \ldots,$$

where $\Omega$ is a fixed vector and $H^{\otimes_a^n}$ is an $n$th antisymmetrized power of $H$ generated by vectors $f_1\Lambda \ldots \Lambda f_n$ obeying the inner product

$$(f_1\Lambda \ldots \Lambda f_n,\ g_1\Lambda \ldots \Lambda g_n) = \det ||(\mathrm{f_j}, \mathrm{g_k})||,$$

$f_j, g_k \in H$. The semigroup of right shifts $S = \{S_t = e^{-td},\ t \geq 0\}$ in $H$ determines the semigroup of unital $*$-endomorphisms $\alpha = \{\alpha_t,\ t \geq 0\}$ on $\mathfrak{A}(H)$ by the formula

$$\alpha_t(a^*(f_1)...a^*(f_n)a(g_1)...a(g_m)) = a^*(S_tf_1)...a^*(S_tf_n)a(S_tg_1)...a(S_tg_m).$$

It is possible to extend $\alpha$ to the algebra $B(F(H))$ of all bounded operators in $F(H)$. Let us continue the semigroup $S$ up to the semigroup of right shifts $\hat{S} = \{\hat{S}_t,\ t \geq 0\}$ in $F(H)$ as follows

$$\hat{S}_t(f_1\Lambda \dots \Lambda f_n) = S_t f_1 \Lambda \dots \Lambda S_t f_n,\ \hat{S}_t\Omega = \Omega.$$

Along with the semigroup $\alpha$ one can consider the semigroup of non-unital $*$-endomorphisms $\hat{\alpha} = \{\hat{\alpha}_t,\ t \geq 0\}$ acting in $B(F(H))$ as follows:

$$\hat{\alpha}_t(x) = \hat{S}_t x \hat{S}_t^*,\ x \in B(F(H)),$$

possessing the unbounded generator

$$(\psi, \hat{\mathcal{L}}(x)\phi) = (\hat{d}\psi, x\phi) + (\psi, x\hat{d}\phi),$$

where $\psi$, $\phi$ are differentiable functions from $F(H)$ and

$$\hat{d}(f_1\Lambda \dots \Lambda f_n) = \sum_{j=1}^{n} f_1\Lambda \dots \Lambda f_{j-1}\Lambda df_j \Lambda f_{j+1}\Lambda \dots \Lambda f_n,\ \hat{d}\Omega = 0.$$

Put $\mathbf{f} = f_1\Lambda \dots \Lambda f_n$ and $\mathbf{f}_{\backslash j} = f_1\Lambda \dots \Lambda f_{j-1}\Lambda f_{j+1}\Lambda \dots \Lambda f_n$, $f_j \in H$. We show that the generator $\mathcal{L}$ of the semigroup $\alpha$ is a singular perturbation of $\hat{\mathcal{L}}$ determined by

$$(\mathbf{f}, \mathcal{L}(x)\mathbf{g}) = (\mathbf{f}, \hat{\mathcal{L}}(x)\mathbf{g}) + \Phi[\mathbf{f}, x\mathbf{g}],$$

where the singular form reads as

$$\Phi[\mathbf{f}, \mathbf{g}] = \sum_{j,k} (-1)^{j+k} \overline{f_j(0)} g_k(0) (\mathbf{f}_{\backslash j}, \mathbf{g}_{\backslash k}).$$

**T. Andreeva.** *On the surjectivity of convolution operators on holomorphic weighted spaces in bounded convex domains.*

Let $G$ be a domain in $\mathbb{C}$ and $H(G)$ the space of all holomorphic functions in $G$. For a continuous function (a weight) $v : G \to \mathbb{R}$, define the Banach space

$$H_v(G) := \Big\{ f \in H(G) : ||f||_v := \sup_{z\in G} |f(z)|e^{-v(z)} < \infty \Big\}.$$

For an increasing sequence of weights $V = (v_n)$ define the inductive limit $\mathcal{V}H(G) := \operatorname{ind} H_{v_n}(G)$. Let $\mu$ be an analytic functional on $\mathbb{C}$ carried by a convex compact set $K$. Under restrictions on weight sequence used by V.V. Napalkov [1], we study the continuity and surjectivity problem of the convolution operator $\mu * f(z) : f \mapsto \mu_w f(z+w)$ that maps $\mathcal{V}H(G+K)$ into (onto) $\mathcal{V}H(G)$. We establish the surjectivity criteria for convolution operator in terms of its Laplace (Fourier–Borel) transform $\hat{\mu}(\zeta) := \mu_z e^{\langle z \cdot \rangle}$ via an appropriate description of functional weighted spaces that are conjugated to $\mathcal{V}H(G+K)$ and $\mathcal{V}H(G)$. The research was supported by the Presidential Program for Support of Young Candidates of Sciences under grant MC-1056.2018.1, agreement 075-02-2018-433.

[1]. Napalkov V. V. Spaces of analytic functions of prescribed growth near the boundary, Math. USSR-Izv., **30:2** (1988), 263–281.

**M. Balashov.** *Proximal smoothness of the real Stiefel manifold.*

Let $n, k \in \mathbb{N}$ and $k \le n$. The real Stiefel manifold is

$$S_{n,k} = \{X \in R^{n\times k} \mid X^T X = I_k\}.$$

The Stiefel manifold is a very popular object in optimization [1]. In the space of matrices $R^{n\times k}$, consider the inner product $\langle X, Y\rangle = \operatorname{tr} X^T Y$ and the norm $\|X\| = \sqrt{\langle X, X\rangle}$. Recall that a closed set $A$ in a Banach space is proximally smooth with constant $R > 0$ [2] if the distance function $\varrho_A(x) = \inf_{a\in A} \|x - a\|$ is continuously Frechet differentiable on the set $U_A(R) = \{x \mid 0 < \varrho_A(x) < R\}$. We claim that $S_{n,k}$ is proximally smooth set with constant $R \ge \dfrac{2}{\sqrt{k^2+3k}}$.

On the base of the last inequality some variants of the gradient projection algorithms can be applied for minimization of a smooth function on the Stiefel manifold. Supported by RSF 16-11-10015.

[1] Edelman Alan, Arias Tomas A. and Smith Steven T. The Geometry of Algorithms with Orthogonality Constraints, Siam J. Matrix Anal. Appl. **20:2** (1998) 303–353.

[2] F.H. Clarke, Yu. S. Ledyaev, R.J. Stern, P.R. Wolenski, Nonsmooth analysis and control theory, Springer-Verlag New-York Inc., 1998.

**A. Baranov.** *Spectral theory of rank one perturbations of normal operators.*

We use a functional model for rank one perturbations of compact normal operators to study their spectral properties. In particular, we discuss completeness of a rank one perturbation and of its adjoint as well as the possibility of the spectral synthesis, i.e., reconstruction of any invariant subspace from the eigenvectors it contains.

**E. Doubtsov.** *Clark measures in several variables.* (Joint work with A.B. Aleksandrov.)

Let $\mathbb{D}$ denote the unit disc of $\mathbb{C}$ and let $\mathcal{D}$ denote the polydisc $\mathbb{D}^n$ or the unit ball of $\mathbb{C}^n$, $n \geq 2$. Given a holomorphic function $\varphi : \mathcal{D} \to \mathbb{D}$, we study the corresponding family $\sigma_\alpha[\varphi]$, $\alpha \in \partial\mathbb{D}$, of Clark measures. If $\varphi$ is an inner function, then we introduce and investigate related isometric operators $U_\alpha$ mapping analogs of model spaces into $L^2(\sigma_\alpha)$, $\alpha \in \partial\mathbb{D}$.

**V. Eiderman.** *A "rare" plane set with Hausdorff dimension* 2. (Joint work with Michael Larsen.)

We prove that for every at most countable family $\{f_k(x)\}$ of real functions on $[0, 1)$ there is a single-valued real function $F(x)$, $x \in [0, 1)$, such that the Hausdorff dimension of the graph $\Gamma$ of $F(x)$ equals 2, and for every $C \in \mathbb{R}$ and every $k$, the intersection of $\Gamma$ with the graph of the function $f_k(x) + C$ consists of at most one point.

**K. Fedorovskiy.** *Approximation by simplest fractions and Chui conjecture.* (Joint work in progress with E. Abakumov and A. Borichev.)

We will deal with the Chui conjecture related to approximation of functions by simplest fractions. We will consider this conjecture in a special scale of weighted Bergman spaces, and show that there is a critical point on this scale which governs the respective approximation properties.

**H. Hedenmalm.** *Planar orthogonal polynomials, random normal matrices and arithmetic jellium.*

The asymptotical analysis of orthogonal polynomials goes back to the 19th century. Interest was strong in the 1920s with prominent work of Szegő and Carleman. More recently, in very special 1D settings, it was understood how to set up Riemann-Hilbert problems to study orthogonal polynomials with respect to exponentially varying weights. But there was no analogue in 2D, and it was believed that the problem would be untractable. Here, we show that it is indeed possible to obtain an asymptotic formula under reasonable smoothnesss and topological assumptions. We apply the obtained asymptotics to obtain boundary universality of the blow-up process associated with the eigenvalues of random normal matrices for smooth spectral boundaries. We also introduce a new process, which we call arithmetic jellium. It is obtained by using the correlation kernel where only the orthogonal polynomials degrees in an arithmetic progression are allowed. This process exhibit hidden symmetries induced by representation theory (the $SU(1,1)$ metaplectic Fock space representation).

**O. Ivrii.** *Homogenization of random quasiconformal mappings and random Delauney triangulations.* (Joint work with Vlad Markovic.)

In this talk, I show that a random quasiconformal mapping is close to an affine mapping, while a circle packing of a random Delauney triangulation is close to a conformal map.

**V. Ivrii.** *Complete spectral asymptotics for periodic and almost periodic perturbations of constant operators and Bethe–Sommerfeld conjecture in semiclassical settings.*

Under certain assumptions, we derive a complete semiclassical asymptotics of a spectral function of the constant coefficient scalar operator perturbed by almost periodic "smaller" operator. In particular, a complete semiclassical asymptotics of the integrated density of states also holds.

Bethe–Sommerfeld conjecture in semiclassical settings holds under similar assumptions for periodic perturbations.

**E. Kalita.** *Estimates of some selfadjoint operators in the neighborhood of $L_2$.*

We consider some class of operators, selfadjoint in (vectorial) $L_2(\mathbb{R}^n)$ and defined in a neighborhood of $L_2$ in the scale of Lebesgue spaces. Typical representative of our class be the operator $D\Delta^{-1}\mathrm{div}$.

The estimate $M+c|p-2|$ for the norm of operator in $L_p$, $|p-2|<\varepsilon$, $M$ is the norm of operator in $L_2$, comes just from Riesz-Thorin interpolation theorem. Such estimate takes place in a far more general situation, and in general no higher smoothness is possible.

We establish the estimate $M+c|p-2|^2$, $\;|p-2|<\varepsilon$, for our class of operators. The same results are established in spaces $L_2$ with Muckenhoupt weights.

**V. Kapustin.** *Hitt's theorem for the half-plane and rational interpolation.*

Many classical problems admit reformulations in terms of kernels of Toeplitz operators on the Hardy space $H^2$ in the upper half-plane. For instance, the condition from the Beurling–Malliavin theorem is equivalent to the fact that the kernel of the Toeplitz operator with symbol $\bar{\theta}B$ is zero, where $\theta(z) = \exp(iaz)$ and $B$ is an infinite Blaschke product. We use the analog for the half-plane of Hitt's theorem about nearly invariant subspaces and our recent results on Toeplitz kernels on the disk to obtain an equivalent condition in terms of rational interpolation.

**B. Khabibullin.** *Affine balayage and zeros of holomorphic functions.*

Let $D \subset \mathbb{R}^d$ be a domain, and $S \Subset D$. Let $V$ be a class of Borel measurable functions on $D\backslash S$. We say that a measure $\mu \in \mathrm{Meas}^+(D\backslash S)$ is an *affine $V$-balayage* of a measure $\upsilon \in \mathrm{Meas}^+(D \setminus S)$ *outside* $S$ and write $\upsilon \preccurlyeq_{S,V} \mu$ if there exists a constant $C \in \mathbb{R}$ such that

$$\int_{D\backslash S} v \,\mathrm{d}\upsilon \leq \int_{D\backslash S} v \,\mathrm{d}\mu + C \quad \textit{for all } v \in V.$$

If $C = 0$ for all $v \in V$ in (1), then it is *usual* classical $V$-balayage, or $V$-sweeping out. For a subset $S \subset \mathbb{R}^d$, we denote by $\mathrm{har}(S)$ and $\mathrm{sbh}(S)$ the classes of all *harmonic* (affine for m = 1) and *subharmonic* (locally convex for $m = 1$) functions on an open $O \supset S$, respectively.

**Main Theorem.** *Let $D$ be a non-empty domain in $\mathbb{R}^d$ with non-polar boundary $\partial D$ and a non-empty subset $S_0 \Subset D$, $M \in \mathrm{sbh}(D)$ be a continuous function on $D$ with the Riesz measure $\mu_M \in \mathrm{Meas}^+(D)$, and $u \not\equiv -\infty$ be a subharmonic function on $D$ with the Riesz measure $\upsilon_u \in \mathrm{Meas}^+(D)$. Then the following two "subharmonic" statements are equivalent:*

$(s_1)$ *There is $v \in \mathrm{sbh}(D)$ such that $v \equiv -\infty$ and $u + v \leq M$ on $D$.*

($s_2$) *The measure $\mu_M$ is an affine $V$-balayage of the measure $\upsilon_u$ outside a subset $S_0 \Subset D$ for the class*

$$V = \Bigl\{v \in \mathrm{sbh}_0(D \setminus S_0)\colon v \geq 0 \textit{ on } D \setminus S_0,\ \sup_{D\setminus S_0} v \leq 1\Bigr\},$$

$$\mathrm{sbh}_0(D \setminus S_0) := \Bigl\{v \in \mathrm{sbh}(D \setminus S_0)\colon \lim_{D\ni x'\to x} v(x') = 0 \textit{ for all } x \in \partial D\Bigr\}.$$

*If, additionally, our set $S_0$ is connected, and there is $S \subset D$ such that*

$$S_0 \Subset S, \quad H := \Bigl\{h \in \mathrm{sbh}_0(D \setminus S_0)\colon \sup |h| < +\infty \textit{ on } S \setminus S_0\ \Bigr\},$$

*then the following two "harmonic" statements are equivalent:*

($h_1$) *There is a function $h \in \mathrm{har}(D)$ such that $u + h \leq M$ on $D$ similar ($s_1$).*

($h_2$) *$\mu_M$ is an affine $V$-balayage of $\upsilon_u$ outside $S_0$ for the class*

$$V_{\pm} = \Bigl\{v \in \mathrm{sbh}_0(D \setminus S_0)\colon \sup_{S\setminus S_0} |v| \leq 1,\ \exists\, S_v \Subset D : v\,\Big|_{S_v} \geq 0\Bigr\}.$$

*Let $D$ be a non-empty domain in the complex plane $\mathbb{C}$, $M$ be a subharmonic continuous function on $D$,*

$$\mathit{Hol}(D, M) := \{f \in \mathit{Hol}(D)\colon |f| \leq \exp M \textit{ on } D\}.$$

**Corollary.** *Let a point sequence $Z := \{z\}_{k=1,2,\ldots} \subset D$ have not limit point in $D$. This sequence $Z$ is a zero set, taking into account multiplicity, for a function from $Hol(D, M)$ if and only if there is a constant $C := \mathrm{const}^+_{M,Z,S_0}$ such that for a class $V_\pm$ from ($h_2$) we have*

$$\sum_k v(z_k) \leq \int_{D\setminus S_0} v \,\mathrm{d}\mu_M + C \quad \textit{for all } v \in V\pm.$$

The study was carried out at the expense of the Russian Science Foundation, project No 18-11-00002.

**E. Korotyaev.** *Inverse resonance scattering for the Laplacian on rotationally symmetric manifolds.* (Joint work with H. Isozaki.)

We discuss inverse resonance scattering for the Laplacian on a rotationally symmetric manifold $M$ whose rotation radius is constant outside some compact interval. The Laplacian on M is unitarily equivalent to a direct sum of one-dimensional Schrodinger operators with compactly supported potentials on the half-line. We prove

(1) Asymptotics of counting function of resonances at large radius.
(2) The rotation radius is uniquely determined by its eigenvalues and resonances.
(3) There exists an algorithm to recover the rotation radius from its eigenvalues and resonances.

The proof is based on some non-linear real analytic isomorphism between two Hilbert spaces.

**A. Kuznetsova.** *Local groups and related $C^*$-algebras.* (Joint work with V. Arzumanian and S. Grigoryan.)

Basing on the notion of local topological group, we consider the "local group" $\mathcal{G}$ which is the local topological group with discrete topology and an additional requirement: if the elements $a$ and $b$, $b$ and $c$, and the elements $ab$ and $c$ can be multiplied, then the elements $a$ and $bc$ can be multiplied and $a(bc) = (ab)c$, $a, b, c \in \mathcal{G}$. Local groups arise naturally: for instance, if $\Gamma$ is a group, $\mathfrak{A}$ is a unital $C^*$-algebra, $\pi : \Gamma \longrightarrow \mathfrak{A}$ is a partial representation, then $\mathcal{G} = \{a \in \Gamma : \pi(a) \neq 0\}$ is a local group.

We define the Fell bundle on a local group. Also we suggest a construction of a reduced $C^*$-algebra $C_r^*(P)$ associated with a subset $P$ of a discrete group $\Gamma$. The algebra is generated by partial isometries on $l^2(P)$ and there is a Fell bundle on the local group which is a grading for $C_r^*(P)$. We give examples of $C_r^*(P)$, in particular, an example of $UHF$-algebra which is "generated" by a subset of abelian group.

**L. Maergoiz.** *Analytic continuation methods for multivalued functions of one variable and their application to the solution of algebraic equations.*

The talk discusses several methods of analytic continuation of a multivalued function of one variable given on a part of its Riemann surface in the form of a Puiseux series generated by the power function $z = w^{1/\rho}$, where $\rho > 1/2$ and $\rho \neq 1$. We present a many-sheeted variant of a theorem of G. Pólya describing the relation between the indicator and conjugate diagrams for entire functions of exponential type. The description is based on a construction of V. Bernstein for the many-sheeted indicator diagram of an entire function of order $\rho \neq 1$ and of normal type. The summation domain of a "proper" Puiseux series (a many-sheeted "Borel polygon") is found with the use of a generalization of the Borel method. This result seems to be new even in the case of power series. The theory applies to describe the domains of analytic continuation of Puiseux series representing the inverse functions for the rational ones. As but one consequence we elaborate a new approach to solution of algebraic equations.

[1] Maergoiz L.S. Many-sheeted versions of the Polya–Bernstein and Borel theorems for entire functions of order $\rho \neq 1$ and their appl., Dokl. Math., **97:1** (2018), 42–46.

[2] Maergoiz L.S. Ways of analytic continuation of many-valued function of one variable. Applications to the solution of algebraic equations, Trudy Inst. Mat. Mekh. UrO RAN, **25:1** (2019), 120–135.

**A. Mirotin.** *On spectral representation for multidimensional normal Hausdorff operators.*

Hausdorff operators have originated from some classical summation methods. Now this is an active research field. In the talk, a spectral representation for multidimensional normal Hausdorff operator will be given. We show that normal Hausdorff operator in $L^2(\mathbb{R}^n)$ is unitary equivalent to the operator of multiplication by some matrix-valued function (its matrix symbol) in the space $L^2(\mathbb{R}^n;\mathbb{C}^{2^n})$. Several corollaries will be considered. In particular, the norm and the spectrum of such operators will be described in terms of the symbol.

[1] A. R. Mirotin, Boundedness of Hausdorff operators on Hardy spaces $H^1$ over locally compact groups, J. Math. Anal. Appl., **473**, (2019), 519 – 533.

**A. Mokeev.** *Non-commutative operator graphs generated by orbits of unitary groups.* (Joint work with G.G. Amosov and A.N. Pechen.)

For a Hilbert space $H$, a non-commutative operator graph (operator systems in other terminology) is a linear subspace $\mathcal{V}\subset B(H)$ with the properties

$$A\in\mathcal{V}\Rightarrow A^*\in\mathcal{V};\ I\in\mathcal{V}.$$

The cases interesting for applications are graphs satisfying the Knill-Laflamme condition, that is, graphs $\mathcal{V}$ satisfying equation $P\mathcal{V}P=\mathbb{C}P$ for some orthogonal projection $P\in B(H)$.

Recently, it has been initiated a study of operator graphs generated by resolutions of identity covariant with respect to the action of unitary representation of some group [1]. Several interesting examples could be constructed by such technique [2], [3]. A separate attention will be paid to the infinite dimensional case in which the non-commutative operator graph is generated by the orbits of unitary group giving a solution to the Schroedinger equation describing dynamics of two-mode quantum oscillator [4]. It is shown that this graph satisfies the Knill-Laflamme condition.

[1] G.G. Amosov. On general properties of non-commutative operator graphs, Lobachevskii Journal of Mathematics **39:3** (2018), 304–308.

[2] G.G. Amosov and A. S. Mokeev. On non-commutative operator graphs generated by covariant resolutions of identity. Quantum Information Processing **17:12** (2018).

[3] G.G. Amosov and A. S. Mokeev. On non-commutative operator graphs generated by reducible unitary representation of the Heisenberg-Weyl group, International Journal of Theoretical Physics.

[4] G.G. Amosov, A.S. Mokeev and A.N. Pechen. Non-commutative operator graphs via dynamics of two-mode quantum oscillator (in preparation).

**N. Nikolski.** *Quantitative constraints for signatures of unconditional bases and frames.* (Joint work with A. Volberg.)

We show that $l^2$ weighted signs of any frame in an $L^2$ space with respect to a continuous measure are "well distributed" on the unit circle. In particular, there is no frames and/or Riesz bases sectorial (e.g., positive) on a set of positive measure. Similar results are obtained for unconditional bases in reflexive Banach lattices between $L^\infty$ and $L^1$, in particular in $L^p$, $1 < p < \infty$.

**P. Ohrysko.** *Inversion problem in measure algebras.* (Joint work with Mateusz Wasilewski.)

Let $G$ be a locally compact Abelian group with its dual $\widehat{G}$ and let $M(G)$ denote the Banach algebra of complex-valued measures on $G$. The classical Wiener–Pitt phenomenon asserts that the spectrum of a measure may be strictly larger than the closure of the range of its Fourier–Stieltjes transform. In particular, if $G$ is non-discrete, there exists $\mu \in M(G)$ such that $|\widehat{\mu}(\gamma)| > c > 0$ for every $\gamma \in \widehat{G}$ but $\mu$ is not invertible. In the paper "In search of the invisible spectrum", N. Nikolski suggested the following problem:

Let $\mu \in M(G)$ satisfy $\|\mu\| \leq 1$ and $|\widehat{\mu}(\gamma)| \geq \delta$ for every $\gamma \in \widehat{G}$. What is the minimal value of $\delta_0$ assuring the invertibility of $\mu$ for every $\delta > \delta_0$? What can be said about the inverse (in terms of $\delta$)?

In my talk I show that $\delta_0 = \frac{1}{2}$ is the optimal value for the first question (for non-discrete $G$). Also, I will present a partial solution for the quantitative variant of the problem (second question): if all elements of $G$ (except the unit) are of infinite order then we can control the norm of the inverse for every $\delta > \frac{-1+\sqrt{33}}{8} \simeq 0,593$. This improves the original result of Nikolski: $\delta > \frac{1}{\sqrt{2}} \simeq 0,707$.

**V. Peller.** *Functions of pairs of contractions under perturbations.*

I am going to consider two problems for functions of perturbed pairs of contractions. The first problem deals with commuting contractions. The first main result is that if $(T_1, R_1)$ and $(T_2, R_2)$ are pairs of commuting contractions, then for $p \in [1, \infty]$ and for an arbitrary functions $f$ analytic in the bidisk of Besov class $B^1_{\infty,1}$, the following inequality holds:

$$\|f(T_2, R_2) - f(T_1, R_1)\|_{S_p} \leq \text{const} \cdot \max\{\|T_2 - T_1\|_{S_p}, \|R_2 - R_1\|_{S_p}\}.$$

Here $S_p$ is the Schatten–von Neumann class $S_p$.

The second main result is obtained jointly with A.B. Aleksandrov. It says that if $(T_1, R_1)$ and $(T_2, R_2)$ are pairs of noncommuting contractions, then the same inequality holds for $p \in [1, 2]$ and does not hold for $p > 2$.

**S. Platonov.** *Fourier–Jacobi harmonic analysis and some problems of approximation of funcions on the half-axis in $L_2$ metric.*

We use methods of Fourier–Jacobi harmonic analysis to study problems of the approximation of functions in weighted $L_2$ function spaces on the half-axis $[0, +\infty)$. We prove analogues of Jackson's direct theorem for the moduli of smoothness of arbitrary orders constructed on the basis of Jacobi generalized translations, and we define function spaces of Nikol'skii–Besov type and describe them in terms of the best approximations. As the approximation tool, we use a class of functions with bounded spectrum, that is, a class of functions for which their Fourier–Jacobi transforms are functions with compact support.

**S. Simonov** *Wave models of metric spaces.*

Let $(\Omega, d)$ be a complete metric space. We describe two ways to construct an isometric copy (the so called "wave model") $(\widetilde{\Omega}, \widetilde{d})$ of the space $(\Omega, d)$ (the original) in terms of the lattice theory. For the first model the lattice of open sets $\mathfrak{O}$ of $\Omega$ is used with the order topology. The second model requires a Borel measure $\mu$ on $\Omega$ and a sufficiently large family of open sets $\mathcal{E}$ from $\Omega$ with boundaries of zero measure $\mu$. The lattice of measurable sets is used with the metric topology. This construction has applications to inverse problems of mathematical physics.

**L. Slavin.** *The Helson–Szego theorem and the $A_2$ condition.*

We refine the statement of the Helson–Szego theorem to incorporate the $A_2$-characteristic of the weight and explore the ensuing connections with the Fefferman–Stein decomposition of BMO and the distance in BMO to $L^\infty$.

**D. Stolyarov.** *Martingale models for cancelling and weakly cancelling differential operators.*

It is a common practice to construct probabilistic models for problems in Harmonic Analysis. I will provide a martingale model for a class of differential operators called cancelling operators. These are exactly the operators for which an analog of the Gagliardo–Nirenberg inequality holds. I will also show an interpretation of a larger class of operators called weakly cancelling operators.

**S. Treil.** *Convex body domination and weighted estimates with matrix-valued weights.* (Joint work with F. Nazarov, S. Petermichl and A. Volberg.)

Main motivations for the weighted estimates with scalar and matrix weights come from the theory of stationary random processes, unconditional wavelet bases, and invertibility of Toeplitz operators.

Recent developments in harmonic analysis, especially the powerful technique of domination by sparse operators allow us to obtain new results and to simplify proofs of the known ones. At the first glance it looks like the technique of sparse domination is specifically scalar-valued, and is not applicable to the estimates with matrix weights. However, the idea of *convex body domination*, introduced by F. Nazarov, S. Petermichl, S. Treil and A. Volberg allows one to generalize the sparse domination technique to the weighted settings.

In the talk I discuss the classical (scalar) sparse domination, and its generalization to the vector-valued case, the sparse convex body domination. I'll show how it allows to simplify the weighted estimates, and discuss some open problems.

**I. Vasilyev.** *On the multidimensional Havin–Shamoyan–Carleson–Jacobs theorem.*

In this talk I will compare local boundary smoothness of an analytic function on the unit ball to the smoothness of its modulus. We shall see that, in dimensions 2 and higher, two different conditions imposed on the zeros of the function imply two different decays of its smoothness compared to the smoothness of its absolute value. On top of that, I will give examples of functions showing that some decays are the best possible.

**O. Vinogradov.** *Two-sided estimates for deviations of quasiprojectors.*

Let $U_h f(x) = \sum\limits_{j\in\mathbb{Z}} \left\langle f, \frac{1}{h}\zeta\left(\frac{\cdot}{h}-j\right)\right\rangle \varphi\left(\frac{x}{h}-j\right)$, $\langle f,g\rangle = \int_{\mathbb{R}} f\overline{g}$. Under certain conditions on functions $\varphi$ and $\zeta$, the operators $U_h$ act in $L_p(\mathbb{R})$. For a wide class of operators $U_h$, we obtain two-sided estimates of the form

$$\|f - U_h f\|_p \asymp \omega_r(f,h)_p$$

and

$$\|f - U_h f\|_p \asymp \omega_{r+1}(f,h)_p + h^{-1}\omega_{r+1}(Jf^{(-1)},h)_p,$$

where $J$ is the Hilbert transform, $\omega_s$ are the moduli of continuity.

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