Euler International Mathematical Institute

XXVII St. Petersburg Summer Meeting in Mathematical Analysis

August 6–11, 2018

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PROGRAM

MONDAY, August 6

09:30–10:30 REGISTRATION

10:30–11:15 **A. Borichev.** *Counting zeros of smooth functions.*

Coffee break

11:45–12:30 **A. Sergeev.** *Kähler geometry of universal Teichmüller space.*

Lunch

15:00–15:45 **A. Kostenko.** *Generalized indefinite strings.*

15:50–16:15 **K. Dyakonov.** *Univalent polynomials and Koebe's one-quarter theorem.*

Coffee break

16:40–17:05 **E. Dubtsov.** *Approximation of log-convex weights by integral means of holomorphic functions.*

17:10–17:35 **S. Popenov.** *Interpolation in convex domains by sums of series of exponentials.*

WELCOME PARTY

TUESDAY, August 7

10:00–10:45 **S. Tikhonov.** *New inequalities for multivariate polynomials.*

Coffee break

11:15–12:00 **E. Gluskin.** *On the symplectic capacity of the difference body.*

12:05–12:50 **L. Slavin.** *The BMO-BLO norm of the maximal operator on α-trees.*

Lunch

15:00–15:45 **E. Korotyaev.** *Inverse resonance scattering on rotationally symmetric manifolds.*

15:50–16:15 **B. Khabibullin.** *On zeros of holomorphic functions on the unit disk/ball: non-radial characteristics.*

Coffee break

16:40–17:05 **A. Mirotin.** *Livschits–Krein trace formula for operator monotonic functions on Banach spaces.*

17:10–17:35 **O. Reinov.** *Around Grothendieck's theorem on operators with nuclear adjoints.*

WEDNESDAY, August 8

FREE DAY, BOAT TRIP

THURSDAY, August 9

10:00–10:45 **D. Yakubovich.** *Spectral study of perturbations of normal operators.*

Coffee break

11:15–12:00 **H. Woracek.** *Schatten-class properties of canonical systems.*

12:05–12:50 **S. Treil.** *Matrix weights and finite rank perturbations of self-adjoint operators.*

Lunch

15:00–15:45 **G. Mikhalkin.** *Holomorphic forms in the tropical limit, and the tropical argument principle.*

15:50–16:15 **R. Pruckner.** *Density of the spectrum of Jacobi matrices with power asymptotics.*

Coffee break

16:40–17:05 **D. Rutsky.** *A_1-regularity and boundedness of Calderon–Zygmund operators in Banach lattices.*

17:10–17:35 **A. Mokeev.** *On reducible unitary representations of the circle group and associated non-commutative operator graphs.*

CONFERENCE DINNER

FRIDAY, August 10

10:00–10:45 **S. Favorov.** *Distributions with discrete support, quasicrystals, and almost periodicity.*

Coffee break

11:15–12:00 **S. Denisov.** *Spectral Szego theorem on the real line.*

12:05–12:50 **E. Abakumov.** *Ordered structure for Cauchy-de Branges spaces and Krein-type theorems.*

Lunch

15:00–15:25 **V. Kapustin.** *Kernels of Toeplitz operators and rational interpolation.*

15:30–15:55 **A. Lishanskii.** *On hypercyclic rank one perturbations of unitary operators.*

16:00–16:25 **A. Pyshkin.** *A linear summation method for a finite-diagonal M-basis.*

Coffee break

16:45–17:10 **A. Kuznetsova.** *Some examples of extensions by compact operators.*

17:15–17:40 **E. Kalita.** *Estimates of singular sets for solutions of nonlinear elliptic systems.*

SATURDAY, August 11

10:00–10:45 **H. Hedenmalm.** *Planar orthogonal polynomials, boundary universality, and porous jellium.*

Coffee break

11:15–12:00 **K. Fedorovskiy.** *Lip^m reflection of harmonic functions across boundaries of simple Caratheodory domains in* $\mathbb{R}^n$.

12:05–12:50 **V. Peller.** *Absolute continuity of spectral shift via the Sz.Nagy theorem on unitary dilations.*

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ABSTRACTS

E. Abakumov. *Ordered structure for Cauchy-de Branges spaces and Krein-type theorems.* (Joint work with A. Baranov and Yu. Belov.)

We extend some results of M. G. Krein to the class of entire functions which can be represented as ratios of discrete Cauchy transforms in the plane. As an application, we obtain new versions of de Branges' Ordering Theorem for nearly invariant subspaces in some classes of Hilbert spaces of entire functions.

A. Borichev. *Counting zeros of smooth functions.*

We estimate the number of zeros of quasianalytically smooth analytic functions in the unit disc.

S. Denisov. *Spectral Szego theorem on the real line.* (Joint work with R. Bessonov.)

We characterize even measures on the real line with finite logarithmic integral in terms of Hamiltonian of the corresponding De Branges canonical system.

E. Dubtsov. *Approximation of log-convex weights by integral means of holomorphic functions.*

Let $\mathcal{H}ol(B_d)$ denote the space of holomorphic functions on the unit ball B_d of $\mathbb{C}^d$, $d \geq 1$. Given a log-convex strictly positive weight $w(r)$ on $[0,1)$, we construct a function $f \in \mathcal{H}ol(B_d)$ such that the standard integral means $M_p(f,r)$ and $w(r)$ are equivalent for any $0 < p \leq \infty$. Also, we obtain similar results related to volume integral means.

K. Dyakonov. *Univalent polynomials and Koebe's one-quarter theorem.* (Joint work with D. Dmitrishin and A. Stokolos.)

We discuss polynomial versions of the Koebe one-quarter theorem for univalent functions.

S. Favorov. *Distributions with discrete support, quasicrystals, and almost periodicity.*

We prove the following theorems.

Theorem 1. Let

$$f_j = \sum_{\lambda\in\Lambda_j} p^{(j)}_{\lambda,k} D^k \delta_\lambda, \quad j = 1, 2, \quad k \in (\mathbb{N} \cup 0)^d,$$

be tempered distributions on $\mathbb{R}^d$ with discrete supports Λ_j such that

$$\inf_{\lambda\in\Lambda_j} sup_k |p^{(j)}_{\lambda,k}| > 0$$

and let $\Lambda_1 - \Lambda_2$ be a discrete set. Moreover, let the Fourier transforms $\hat{f}_j$ be discrete complex measures and let each support $\Gamma_j, j = 1, 2$, satisfy the following condition:

$$(*) \quad \exists h < \infty, c > 0 \quad \text{such that} \quad |\gamma - \gamma'| > c \min\{1, |\gamma|^{-h}\} \quad \forall \gamma, \gamma' \in \Gamma.$$

Then Λ_1, Λ_2 are finite unions of translates of a unique full-rank lattice.

Theorem 2. Let μ be a discrete complex measure on $\mathbb{R}^d$ with a uniformly discrete support Λ, $|\mu(\lambda)| \geq c > 0$ for all $\lambda \in \Lambda$, and let the Fourier transform $\hat{\mu}$ be a measure such that $|\hat{\mu}|(B(r)) = O(r^d)$ as $r \to \infty$. Then Λ is a finite union of translates of *several* full-rank lattices.

The proofs of Theorems 1 and 2 are based on a generalization of Wiener's Theorem on Fourier series, and properties of almost periodic distributions and measures. In particular, we prove the following result.

Theorem 3. Let f be a tempered distribution on $\mathbb{R}^d$. If the Fourier transform $\hat{f}$ is a discrete complex measure and the support Γ of $\hat{f}$ satisfies condition $(*)$, then f is an almost periodic distribution.

K. Fedorovskiy. *Lip^m reflection of harmonic functions across boundaries of simple Caratheodory domains in $\mathbb{R}^n$.* (Joint work with P. Paramonov.)

It is planned to consider the problem on Lip^m-continuity of operators of harmonic reflection of functions across boundaries of simple Carathéodory domains in $\mathbb{R}^n$ and the closely related problem on Lip^m-continuity of the Poisson operator in such domains. We will present some new (sharp) necessary and sufficient conditions for Lip^m continuity of both these operators. As a corollaries of these results we will present new sufficient conditions for Lip^m-approximation of functions by harmonic polynomials on boundaries of simple Caratheodory domains in $\mathbb{R}^n$.

E. Gluskin. *On the symplectic capacity of the difference body.*

Symplectic capacities is a wide class of symplectic invariants. This class has the maximal element, it is the cylindrical capacity. Other important example is Hofer-Zehnder capacity. It is intimately related to periodic orbits of Hamiltonian systems. About three years ago Y.Ostrover and the speaker introduced some linear analogues of the cylindrical capacity, and named it linear capacity. They proved that these three quantities are weakly equivalent for a centrally symmetric convex body. Recently it was shown that such equivalence doesn't hold in the class of all convex bodies. The main step in the proof is related to finite dimensional analogues of the Hilbert operator.

H. Hedenmalm. *Planar orthogonal polynomials, boundary universality, and porous jellium.* (Joint work with A. Wennman.)

Strong asymptotics of the orthogonal polynomials was obtained recently and we report on advances based on this result. In particular we discuss boundary universality for random normal matrices.

E. Kalita. *Estimates of singular sets for solutions of nonlinear elliptic systems.*

Let $\mathrm{d}iv^m A(x, D^m u) = 0$ be an elliptic system in domain in $\mathbb{R}^n$ with the natural energy space W_2^m and standard structure conditions. Let the functions $A(x, \xi)$ be smooth. By the classical partial regularity theory, any W_2^m-solution is smooth (say, $C^{m+\epsilon}$) in an *open* set of full measure, and the singular set is of Hausdorff dimension $< n - 2$.

We give explicit estimates for Hausdorff dimension of intersections of singular set with $(n-1)$-dimensional hyperplanes, which depends on the modulus of ellipticity of system. The results are based on the estimates of the norm of vector Riesz transform $D\Delta^{-1}\mathrm{d}iv$ in the weighted space $L_2(\mathbb{R}^n; |x_1|^a)$, $|a| < 1$.

V. Kapustin. *Kernels of Toeplitz operators and rational interpolation.*

The kernel of a Toeplitz operator on the Hardy class H^2 in the unit disk is a nearly invariant subspace of the backward shift operator, and, by D. Hitt's result, it has the form $g \cdot K_\omega$, where ω is an inner function, $K_\omega = H^2 \ominus \omega H^2$, and g is an isometric multiplier on K_ω. We describe the functions ω and g for the kernel of the Toeplitz operator with symbol $\bar{\theta}B$, where θ is an inner function and B is a finite Blaschke product.

B. Khabibullin. *On zeros of holomorphic functions on the unit disk/ball: non-radial characteristics.*

Let M be a subharmonic function with Riesz measure ν_M on the unit disk $\mathbb{D}$ (resp. unit ball $\mathbb{B} \subset \mathbb{C}^n$) in the complex plane $\mathbb{C}$ (resp. in $\mathbb{C}^n$, $n > 1$). Let f be a nonzero holomorphic function on $\mathbb{D}$ (resp. on $\mathbb{B}$) such that f vanish on $\mathsf{Z} \subset \mathbb{D}$ (resp. $\subset \mathbb{B}$), and satisfies $|f| \leq \exp M$ on $\mathbb{D}$ (resp. $\mathbb{B}$). Then restrictions on the growth of ν_M near the boundary of D imply certain restrictions on the distribution of Z. We give a quantitative study of this phenomenon in terms of special non-radial test functions constructed using ρ-trigonometrically convex (resp. ρ-subspherical) functions.

E. Korotyaev. *Inverse resonance scattering on rotationally symmetric manifolds.* (Joint work with H. Isozaki.)

We consider the Laplacian on a rotationally symmetric manifold M. We assume that M is a cylindrical manifold with warped product. We show that the Laplacian has an infinite number of eigenvalues for a specific rotation radius, and has no eigenvalues for another specific rotation radius. Moreover, we discuss the resonances of the Laplacian and we solve the inverse problem in terms of resonances.

A. Kostenko. *Generalized indefinite strings.* (Joint work with J. Eckhardt.)

In this talk, we review the direct and inverse spectral theory for generalized indefinite strings. The interest to this class of spectral problems is dictated by applications to nonlinear completely integrable equations (e.g., the conservative Camassa–Holm flow).

A. Kuznetsova. *Some examples of extensions by compact operators.*

The report is devoted to the operator algebras $C^*_\varphi(X)$ in the case, where they are extensions of the algebra $C(S^1)$ of all continuous functions on the unit circle by compact operators. The starting point is a selfmapping $\varphi : X \longrightarrow X$ on a countable set X with a finite preimage for each point. This mapping generates a directed graph with vertices at the points of the set X and the edges $(x, \varphi(x))$. The algebra $C^*_\varphi(X)$ is generated by the composition operator

$$T_\varphi : l^2(X) \to l^2(X), \quad T_\varphi f = f \circ \varphi.$$

Theorem 1. Let $C^*_\varphi(X)$ contain the algebra $K(l^2(X))$ of all compact operators on $l^2(X)$. Then the following properties are equivalent:
1) $C^*_\varphi(X)$ is an extension of $C(S^1)$ by $K(l^2(X))$;
2) the Fredholm index of T_φ is finite.

Let $\mathfrak{E}$ be the set of irreducible algebras $C^*_\varphi(X)_{\varphi\in\Phi}$ such that index$(T_\varphi) \leq 0$.

Theorem 2. Let $C^*_\varphi(X)$ and $C^*_\psi(X)$ be in $\mathfrak{E}$. Then $C^*_\varphi(X)$ and $C^*_\psi(X)$ are isomorphic if and only if 1) index$(T_\varphi) =$ index(T_ψ) and 2) the mappings φ and ψ simultaneously admit (or do not admit) finite orbits.

We equip the set $\mathfrak{E}$ with the semigroup structure isomorphic to $\mathbb{Z}_+$. Also, we consider different examples of nonisomorphic extensions of $C(S^1)$ by compact operators.

A. Lishanskii. *On hypercyclic rank one perturbations of unitary operators.* (Joint work with A. Baranov and V. Kapustin.)

Recently, S. Grivaux showed that there exists a rank one perturbation of a unitary operator in a Hilbert space which is hypercyclic. Using a functional model for rank one perturbations of singular unitary operators, we give a different construction of hypercyclic rank one perturbation of a unitary operator. In particular, we show that any countable union of perfect Carleson sets on the circle can be the spectrum of a perturbed (hypercyclic) operator.

G. Mikhalkin. *Holomorphic forms in the tropical limit, and the tropical argument principle.* (Joint work with N. Kalinin.)

We study holomorphic forms under the so-called tropical degeneration of Riemann surfaces, i.e. their degeneration to metric graphs (the metric is the only data remaining from the conformal structure in the tropical limit).

It turns out that there are two meaningful ways to pass to the tropical limit for the forms: taking into account only the residues we get a complex-valued 1-form on the graph, while a conventional tropical limit yields a piecewise-linear function on the graph. The tropical argument principle is a compatibilty condition for these two limits.

A. Mirotin. *Livschits–Krein trace formula for operator monotonic functions on Banach spaces.*

We give a simple definition of a spectral shift function for pairs of nonpositive operators on Banach spaces. We prove trace formulas of Lifshitz–Krein type for a perturbation of an operator monotonic (negative complete Bernstein) function of negative and nonpositive operators on Banach spaces induced by nuclear perturbation of an operator argument. The results may be regarded as a contribution to a perturbation theory for Hirsch functional calculus.

A. Mokeev. *On reducible unitary representations of the circle group and associated non-commutative operator graphs.* (Joint work with G. Amosov.)

A non-commutative operator graph $\mathcal{V}$ is closed under operator conjugation and contains the identity subspace in the space of all bounded linear operators on the Hilbert space H. Such objects play a significant role in the quantum error correction theory. Graph $\mathcal{V}$ satisfies the Knill–Laflamme–Viola condition if there exists an orthogonal projection P such that $P\mathcal{V}P = \mathbb{C}P$; we will discuss general properties of such graphs. We will study non-commutative operator graphs generated by a resolution of the identity covariant with respect to the action of a unitary represented compact group. It will be shown that under a restriction on the spectral decompositions of operators of the correspondent representation such non-commutative operator graph satisfies the Knill–Laflamme–Viola condition. Examples based on reducible unitary representations of the circle group are of special interest.

V. Peller. *Absolute continuity of spectral shift via the Sz.Nagy theorem on unitary dilations.* (Joint work with M. Malamud and H. Neidhardt.)

I am going to use the method of double operator integral to establish the Lifshits–Krein trace formula and its analogs for unitary operators, contractions, dissipative operators. First of all, this method allows us to prove that spectral shift is absolutely continuous. To achieve this, we have to start with the case of functions of contractions and (what is absolutely unexpected) use the Sz.Nagy theorem on the absolute continuity of the minimal unitary dilation of completely nonunitary contractions. This allows us to proceed from contractions to unitary, self-adjoint and dissipative operators. The same technique also allows us to describe the maximal class of functions, for which the trace formula is applicable.

S. Popenov. *Interpolation in convex domains by sums of series of exponentials.*

Sufficient conditions for solvability of interpolation problems in convex domains are given in terms of mutual disposition of the set of interpolation nodes and the set of exponentials of exponential series. For some specific sets of nodes, criteria of interpolation are obtained. The interpolation problem is closely related to a generalization of Eidelheit theorem that leads to the following problem: when the set of exponentials gives us a sufficient set for all polynomials of exponentials with exponents from a given set of nodes?

R. Pruckner. *Density of the spectrum of Jacobi matrices with power asymptotics.*

We consider Jacobi matrices J whose off-diagonal (ρ_n) and diagonal (q_n) have power asymptotics

$$\rho_n = n^{\beta_1}\big(x_0 + \frac{x_1}{n} + \mathrm{O}(n^{-2})\big), \quad q_n = n^{\beta_2}\big(y_0 + \frac{y_1}{n} + \mathrm{O}(n^{-2})\big),$$

with $\beta_1 > 1$, $x_0 > 0$ and $y_0 \neq 0$. If $\beta_1 > \beta_2$, we show that J is of type C, or synonymously, in the limit circle case. The counting function of the spectrum of selfadjoint extensions has order $1/\beta_1$, and we obtain bounds for the upper density. The result also holds true in some cases where $\beta_1 = \beta_2$, i.e. where diagonal and off-diagonal are comparable.

A. Pyshkin. *A linear summation method for a finite-diagonal M-basis.*

Azoff and Shehada introduced a complete minimal system which was hereditarily complete but did not admit a linear summation method (1993). We study a class of complete minimal systems $\mathcal{F} = \{f_n\}_{n=1}^{\infty}$ in the Hilbert space which represent a generalisation of the Azoff-Shehada system. We determine the conditions under which $\mathcal{F}$ admits a linear summation method, moreover we investigate the 2-completeness property for $\mathcal{F}$. In order to do that we employ some of the graph theory techniques.

O. Reinov. *Around Grothendieck's theorem on operators with nuclear adjoints.*

In 1955, A.Grothendieck proved that if an operator $T : X \to Y$ in Banach spaces has a nuclear adjoint, then T is nuclear provided that X^* has the approximation property. It was shown by T. Figiel and W. B. Johnson (1973) that the assumption on X^* is essential. A generalization of Grothendieck's theorem was obtained in 1987 by E. Oja and O. Reinov: The conclusion of the theorem is true if Y^{***} has the approximation property (and the assumption posed on Y^{***} is also essential). A "revised" (more general) version of the theorem was formulated and proved in 2012 by E. Oja for weak* continuous operators from a dual Banach space into an arbitrary Banach space. She considered even the cases of α-nuclear operators for tensor norms α. We present here, among other results, a "revised" version of Grothendieck's theorem in the cases of α-nuclear operators for projective tensor quasi-norms α.

D. Rutsky. *A_1-regularity and boundedness of Calderon–Zygmund operators in Banach lattices.*

Let X be a Banach space of measurable functions on $\mathbb{R}^n$ (e.g., a variable exponent Lebesgue space $L_{p(\cdot)}$), and let T be a Calderon–Zygmund operator with a certain nondegeneracy property (e.g., a Riesz transform). We show that T is bounded in X if and only if the Hardy–Littlewood maximal operator M is simultaneously bounded in both X and its dual X'. The "only if" part of the proof combines a variety of techniques based on the study of Muckenhoupt A_p-majorants in Banach lattices to achieve full generality, and in particular the following interesting property: M is bounded in both X and X' if and only if M is bounded in $X(\ell^p)$ with some $1 < p < \infty$.

A. Sergeev. *Kähler geometry of universal Teichmüller space.*

At the moment we cannot say that there is a well-developed theory of infinite-dimensional complex manifolds. So it is important to have different examples of such manifolds. One of them is given by the universal Teichmüller space. In our talk we shall present main complex geometric features of this remarkable infinite-dimensional manifold. The universal Teichmüller space $\mathcal{T}$ is the space of normalized quasisymmetric homeomorphisms of the unit circle S^1, i.e. orientation-preserving homeomorphisms of S^1, extending to quasiconformal maps of the unit disk Δ and fixing three points on S^1. It is a complex Banach manifold with the complex structure provided from Bers embedding of $\mathcal{T}$ into the complex Banach space of holomorphic quadratic differentials in a disk. The name of $\mathcal{T}$ is motivated by the fact that all classical Teichmüller spaces $T(G)$, associated with compact Riemann surfaces, are contained in $\mathcal{T}$ as complex subspaces. Another important subspace of $\mathcal{T}$ is given by the space $\mathcal{S}$ of normalized orientation-preserving diffeomorphisms of S^1. The space $\mathcal{S}$ is a Kähler Frechet manifold provided with a symplectic structure compatible with the complex structure of

$\mathcal{S}$. We construct a Grassmann realization of $\mathcal{T}$ by embedding it into the Grassmann manifold of a Hilbert space which coincides with the Sobolev space $V = H_0^{1/2}(S^1, \mathbb{R})$ of half-differentiable functions on the circle. This embedding realizes the group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of S^1 as a subgroup of symplectic group $\mathrm{Sp}(V)$. It also defines an embedding of $\mathcal{T}$ into the space of complex structures on V compatible with symplectic structure. The latter space may be considered as an infinite-dimensional Siegel disk.

L. Slavin. *The BMO-BLO norm of the maximal operator on α-trees.* (Joint work with A. Osękowski and V. Vasyunin.)

It has long been known that maximal functions map BMO into BLO (bounded lower oscillation), but the exact operator norm has not been determined for any maximal basis. We obtain the explicit upper Bellman function for the natural dyadic maximal operator (the one without the absolute value in the average) acting from $\mathrm{BMO}^d(\mathbb{R}^n)$ into $\mathrm{BLO}(\mathbb{R})$. As a consequence, we show that the BMO $\to$ BLO norm of this operator equals 1 for all n. The result is a partial corollary of a theorem for α-trees, which generalize dyadic lattices. The Bellman function in this setting exhibits an interesting quasi-periodic structure depending on α, but also allows a majorant independent of α, hence the dimension-free constant. Explicit extremizing sequences are constructed to demonstrate sharpness.

S. Tikhonov. *New inequalities for multivariate polynomials.* (Joint work with F. Dai, Yu. Kolomoitsev, A. Prymak, and V. Temlyakov.)

In this talk I plan to discuss new Remez and Hardy–Littlewood–Nikolskii type inequalities for multivariate trigonometric polynomials. Several applications will be given.

S. Treil. *Matrix weights and finite rank perturbations of self-adjoint operators.* (Joint work with C. Liaw.)

Matrix-valued measures provide a natural language for the theory of finite rank perturbations. Two weight estimates with matrix weights appear naturally in this context. In the talk I discuss some recent results, in particular the Aronszajn–Donoghue theory about the mutual singularity of the singular parts of the spectrum. While simple direct sum type examples would indicate that such a theory is impossible for the scalar spectral measures, it holds if one introduces the notion of vector mutual singularity of matrix measures. Moreover, for the scalar spectral measures, the mutual singularity holds for almost all perturbations.

I'll also discuss the Aleksanrov's disintegration theorem for matrix Clark measures, as well as a simple proof of the Kato–Rosenblum theorem.

H. Woracek. *Schatten-class properties of canonical systems.* (Joint work with R. Romanov.)

Consider a two-dimensional canonical system

$$y'(x) = zJH(x)y(x), \quad x \in [a,b),$$

where H is a locally integrable function taking positive semidefinite 2×2-matrices as values, z is a complex parameter (the eigenvalue parameter), and J is the signature matrix $J = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$.

The task is to determine whether resolvents of the associated differential operator belong to a given symmetrically normed ideal. Equivalently, we wish to understand the asymptotic behaviour of the spectrum of this operator.

We complete this task for ideals of a particular (fairly general) form. The solution proceeds through two stages. First, we show that membership in an ideal depends only on the diagonal entries of H, and second we characterise membership of resolvents in an ideal for diagonal Hamiltonians. Our results are sufficiently general to cover growth scales familiar from complex analysis. For example, we can determine the convergence exponent provided it is > 1 and decide whether we have finite or minimal type w.r.t. a proximate order corresponding to an order > 1.

D. Yakubovich. *Spectral study of perturbations of normal operators.*

Let N be a bounded normal operator on a separable Hilbert space, whose scalar spectral measure is absolutely continuous with respect to the planar Lebesgue measure. We will discuss the spectral nature of the perturbation $T = N + K$, where K is a sufficiently "smooth" compact operator. We introduce the perturbation operator-function $\Psi(\lambda)$ of T, defined everywhere in $\mathbb{C}$ and use it to establish a quotient model for T, constructed in terms of certain vector-valued Sobolev classes of functions. Various consequences of this representation for the spectral theory of the operators under consideration are discussed. Many of these results have been obtained by the speaker long ago, but I will also speak about our recent joint work with Mihai Putinar, where we try to get rid of all assumptions on the spectral type of N. The case of a diagonalizable operator N has been studied in a series of papers by C. Foias and his coauthors.