Euler International Mathematical Institute

XXVI St. Petersburg Summer Meeting in Mathematical Analysis

June 25-30, 2017

The conference is supported by the Russian Science Foundation, grant No 14-41-00010.

SUNDAY, June 25

09:30–10:30 REGISTRATION

10:30–11:15 **E. Dubtsov.** *Volterra type operators on growth Fock spaces.*

Coffee break

11:45–12:30 **L. Slavin.** *Dimension-free estimates for semigroup BMO and related classes.*

12:35–13:00 **P. Zatitskii.** *On a mysterious coincidence between two extremal problems.*

Lunch

15:00–15:25 **K. Dyakonov.** *A free interpolation problem in a subspace of H^∞.*

15:30–15:55 **S. Ivanov.** *Exponential bases and control of a multichannel system.*

Coffee break

16:15–16:40 **A. Vasin.** *A T(P) theorem in Campanato spaces on domains.*

16:45–17:10 **O. Reinov.** *On symmetry of spectra of linear operators in Banach spaces.*

17:10 WELCOME PARTY

MONDAY, June 26

10:00–10:45 **S. Treil.** *Clark model in general situation.*

Coffee break

11:15–12:00 **P. Paramonov.** *Uniform harmonic approximation on plane compacts and logarithmic capacity.*

12:05–12:50 **A. Poltoratski.** *Toeplitz Order.*

Lunch

15:00–15:25 **S. Popenov.** *Interpolation in convex domains by sums of series of exponentials.*

15:30–15:55 **A. Gasparyan.** *Generalizations of classical identities and inequalities to multidimensional determinants.*

16:00–16:25 **S. Platonov.** *An analog of a Titchmarsh theorem for the Fourier transform on locally compact Vilenkin groups.*

Coffee break

16:45–17:10 **V. Riazanov.** *On the boundary value problems in the plane. An alternative approach.*

17:15–17:40 **I. Kayumov.** *On a powered Bohr inequality.*

TUESDAY, June 27

10:00–10:45 **N. Nikolski.** *Completeness of dilated functions. The case of binomials.*

Coffee break

11:15–12:00 **K. Fedorovskiy.** *Approximative properties of solutions of second order elliptic systems.*

12:05–12:50 **Valentin Napalkov.** *On an H. Shapiro problem in the space of entire functions.*

Lunch

15:00–15:25 **Valerii Napalkov.** *Application of orthosimilar decomposition systems for solving the Newman–Shapiro problem for the Bargmann-Fock space.*

15:30–15:55 **S. Novikov.** *Geometry of frames.*

16:00–16:25 **V. Goryainov.** *A semigroup of holomorphic self-mappings of a strip.*

Coffee break

16:45–17:10 **A. Mirotin.** *On a class of globally operator monotone functions of several variables.*

17:15–17:40 **N. Rautian, V. Vlasov.** *Spectral analysis of functional differential equations with unbounded operator coefficients.*

WEDNESDAY, June 28

18:40–21:00 BOAT TRIP from Pesochnaya, 10 to Mytninskaya, 1.

THURSDAY, June 29

10:00–10:45 **E. Korotyaev.** *Resonances for Stark operators.*

Coffee break

11:15–12:00 **M. Malamud.** *Trace formulas for pairs of m-dissipative operators and contractions.*

12:05–12:50 **R. Zarouf.** *Schäffer's conjecture, Fourier coefficients of Blaschke products and Jacobi polynomials with varying parameters.*

Lunch

15:00–15:25 **H. Bommier-Hato.** *De Branges spaces and Fock spaces.*

15:30–15:55 **E. Kalita.** *Higher derivatives of solutions for elliptic PDEs with discontinuous coefficients.*

16:00–16:25 **G. Amosov.** *On constructing quantum anticliques for the noncommutative operator graphs generated by generalized Pauli matrices.*

Coffee break

16:45–17:10 **N. Rastegaev.** *On spectral asymptotics of the Neumann problem for the Sturm-Liouville equation with arithmetically self-similar singular weight.*

17:15–17:40 **M. Mazalov.** *An L_1-estimate for Calderon's commutators, and some applications.*

18:00 CONFERENCE DINNER at the Euler Institute.

FRIDAY, June 30

10:00–10:45 **A. Comech.** *Spectral stability of small amplitude solitary waves in the nonlinear Dirac equation.*

Coffee break

11:15–12:00 **V. Peller.** *Multiple operator integrals and functions of triples of noncommuting self-adjoint operators.*

12:05–12:50 **H. Hedenmalm.** *Bloch functions, asymptotic variance, and geometric zero packing.*

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ABSTRACTS

G. Amosov. *On constructing quantum anticliques for the noncommutative operator graphs generated by generalized Pauli matrices.* (Joint work with A. S. Mokeev.)

A linear space $\mathcal{V}$ consisting of operators in a Hilbert space H is said to be *a noncommutative operator graph* if the identity operator is in $\mathcal{V}$, and $A \in \mathcal{V}$ implies $A^* \in \mathcal{V}$. The orthogonal projection P in H is said to be a quantum anticliques if

$$\dim P\mathcal{V}P = 1.$$

We will discuss the problem of existence of quantum anticliques for noncommutative operator graphs generated by generalized Pauli matrices.

H. Bommier-Hato. *De Branges spaces and Fock spaces.*

An Hermite–Biehler function E determines the de Branges space $\mathcal{H}(E)$, endowed with the norm $\|F\|_E^2 = \int_{\mathbb{R}} |\frac{F(x)}{E(x)}|^2 dx$. For a measurable weight $W : \mathbb{C} \to [0, +\infty)$, one defines the Fock space

$$\mathcal{F}_W = \left\{ f \in Hol(\mathbb{C}) : \ \|F\|_W^2 = \int_{\mathbb{C}} |f(z)W(z)|^2 dz < +\infty \right\}.$$

We give sufficient/necessary conditions for the spaces $\mathcal{H}(E)$ and $\mathcal{F}_W$ to coincide, with equivalent norms.

A. Comech. *Spectral stability of small amplitude solitary waves in the nonlinear Dirac equation.* (Joint work with N. Boussaid.)

In this talk, we will consider solitary waves $\phi_\omega(x)e^{-\mathrm{i}\omega t}$ in the nonlinear Dirac equation with scalar self-interaction ("Soler model"), proving that, in the case of the "charge-subcritical" nonlinearity, small amplitude solitary waves are *spectrally stable*: that is, linearization at a particular solitary wave contains no eigenvalues with positive real part. An important step in the stability analysis is the proof of the absence of bifurcations of nonzero-real-part eigenvalues from the embedded threshold points at $\pm 2m\mathrm{i}$. Our approach is based on constructing a new family of exact bi-frequency solitary wave solutions $\phi_\omega(x)e^{-\mathrm{i}\omega t} + \chi_\omega(x)e^{\mathrm{i}\omega t}$ in the Soler model, on using this family to determine

the multiplicity of $\pm 2\omega$i eigenvalues of the linearized operator, and on the analysis of the behaviour of "nonlinear eigenvalues": characteristic roots of holomorphic operator-valued functions studied by M. Keldysh. The talk is based on [1].

1. N. Boussaid and A. Comech, Spectral stability of weakly relativistic solitary waves of the Dirac equation with the Soler-type nonlinearity, arXiv:1705.05481 (2017).

E. Dubtsov. *Volterra type operators on growth Fock spaces.* (Joint work with E. Abakumov.)

Using results related to approximation by entire maps, we investigate Volterra type and weighted composition operators defined on the growth Fock spaces.

K. Dyakonov. *A free interpolation problem in a subspace of H^∞.*

We study a free interpolation problem for a subspace of H^∞, namely, for the star-invariant subspace K_B^∞ associated with an interpolating Blaschke product B.

K. Fedorovskiy. *Approximative properties of solutions of second order elliptic systems.*

It is planned to discuss several problems of uniform and C^m-approximation of functions by solutions of second order homogeneous elliptic systems on compact sets in the plane. These problems have been studied in details in the case of skew symmetric systems, that is, in the case of second order elliptic equations with constant complex coefficients. We plan to present new results for general systems and to discuss the similarities and differences between skew symmetric case and the general one.

A. Gasparyan. *Generalizations of classical identities and inequalities to multidimensional determinants.*

Many classical inequalities are of determinantal nature. The list of such inequalities contains, for example, the Newton's inequalities involving symmetric means, the Gram's inequalities, in particular, the Cauchy–Bunyakovsky–Schwarz inequality, the Chebyshev, Gruss, Ostrovski, and a plenty of other inequalities obtained by their generalizations, modifications and combinations.

We are interested in more general inequalities related to multidimensional determinants, instead of 2-dimensional ones. So, we present generalizations of the above-mentioned classical results to a series of multidimensional determinantal inequalities. The proofs are based on determinantal identities, which extend the classical Cauchy–Binet identity to multidimensional matrices and determinants. Also, we discuss applications of the results obtained.

V. Goryainov. *A semigroup of holomorphic self-mappings of a strip.*

Given a strip, symmetric with respect to the real axis, we study the semigroup generated by the operation of composition on holomorphic self-mappings of the strip. The mappings under consideration have bounded deviation from the identity transformation on the real axis. We obtain distortion theorems. Also, we study the infinitesimal structure of the semigroup.

S. Ivanov. *Exponential bases and control of a multichannel system.*

We consider a system consisting of K coupled acoustic channels with different sound velocities c_j. The channels are interacting at every point via the pressure and its time derivatives. Using the moment approach and the theory of exponential families with vector coefficients, we establish the following controllability results:

The system is exactly controllable if

- (i) the control u_j in the j-th channel acts more than the double 'optical' length of this channel;
- (ii) all controls u_j act more than or equal to the double maximal optical length of the channels.

E. Kalita. *Higher derivatives of solutions for elliptic PDEs with discontinuous coefficients.*

We consider elliptic equations and systems

$$\operatorname{div}^m(A(x)D^m u) = f(x), \quad x \in \mathbb{R}^n,$$

under the following assumptions: the matrix of coefficients A is in L_∞, the structure conditions are standard, and the energy space is W_2^m. If $DA \in L_n$, then clearly $D^{m+1}u \in L_2$. However, the typical discontinuity $x/|x|$ just fails

to be in W_n^1. We treat general discontinuities of this type (including dense discontinuities, like in Souček and John–Malý–Stará examples of solutions with dense set of discontinuities) by means of the dual Morrey spaces. We discuss estimates for the higher derivatives $D^{m+\alpha}u$ and for lower ones $D^{m-\alpha}u$. Also, we consider the solvability problem in $W_2^{m-\alpha}$. The results are new even for the second-order equations.

I. Kayumov. *On a powered Bohr inequality.* (Joint work with S. Ponnusamy.)

We are going to describe our method to solve a problem formulated by P. B. Djakov and M. S. Ramanujan in 2000. The problem is related to the powered Bohr radius. Also, we give several applications, including an asymptotically sharp form of one of the results by Djakov and Ramanujan.

E. Korotyaev. *Resonances for Stark operators.*

We consider the Stark operator perturbed by a compactly supported potential (of a certain class) on the real line. We solve the following problems:

(a) find asymptotics of the counting function;
(b) determine the forbidden domain for resonances;
(c) find asymptotics of resonances at high energy;
(d) give the trace formula in terms of resonances only;
(e) prove that all resonances determine the potential uniquely.

M. Malamud. *Trace formulas for pairs of m-dissipative operators and contractions.* (Joint work with H. Neidhardt and V. Peller.)

We will discuss Krein type trace formulas for pairs of two resolvent comparable m-dissipative operators as well as for pairs of contractions with trace class difference. In particular, it will be shown that Krein-type formula remains valid for pairs of contractions and any operator Lipschitz function analytic in the unit disk. The talk is based on works [1] and [2].

1. M. Malamud, H. Neidhardt, Trace formulas for additive and non-additive perturbations, Advances in Math. **274** (2015), 736–832. 2. M. Malamud, H. Neidhardt, V. Peller, Analytic operator Lipschitz functions in the disk and a trace formula for functions of contractions, preprint.

M. Mazalov. *An L_1-estimate for Calderon's commutators, and some applications.*

For a classical Calderon's commutator

$$F(z) = v.p. \int_\Gamma \frac{\psi(\tau) - \psi(x)}{(t-z)^2} f(t)dt, \tag{1}$$

where $z = x + iy$ and $t = \tau + i\nu \in \mathbb{C}$, Γ is a Lipschitz graph $y = g(x)$ with $|g'| \leq 1$, we obtain the following estimate on Γ: $\|F\|_{L_1} \leq A\|f\|_{L_2}\|\psi'\|_{L_2}$.

Also, we consider applications of (1) to extremal problems and to estimates of some capacities, and we discuss certain generalizations.

A. Mirotin. *On a class of globally operator monotone functions of several variables.*

Let U be an open subset of $\mathbb{R}^n, \alpha \in \mathbb{R}$. We say that a continuous positive function f defined on U belongs to the class $Q^\alpha(U)$ if there exists a number γ and a positive measure τ on $\mathbb{R}^n_+$ such that, $\forall z \in U$, $1/f(z) = \gamma + S^\alpha\tau(z)$ where $S^\alpha\tau$ is the n-dimensional generalized Stieltjes transform of τ.

Definition [1, 2]. Let U be an open subset of $\mathbb{R}^n$, and let f be a real-valued continuous function on U. A function f is said to be globally operator monotone on U if, for all collections A and B of n pairwise commuting bounded self-adjoint operators in a Hilbert space such that $\sigma(A) \cup \sigma(B) \subset U$, the inequalities $A_j \leq B_j, j = 1, \ldots, n$, imply $f(A) \leq f(B)$.

Theorem. If $f \in Q^\alpha((0, +\infty)^n)$ for some $\alpha \in (0, 1/n)$, then f is a globally operator monotone function on $(0, +\infty)^n$.

1. J. Agler, J. E. McCarthy, and N. J. Young, Ann. of Math. (2) **176** (2012), 1783–1826.

2. J. Agler, J. E. McCarthy, and N. J. Young, arXiv: 1009.3921 (2013).

Valentin Napalkov. *On an H. Shapiro problem in the space of entire functions.*

We deal with a problem formulated by H. Shapiro for differential operators in the space of entire functions. The problem was solved earlier only in special cases.

Valerii Napalkov. *Application of orthosimilar decomposition systems for solving the Newman–Shapiro problem for the Bargmann–Fock space.*

The report will deal with orthosimilar decomposition systems in Hilbert spaces of analytic functions. Using a new approach, we propose a method to solve the Newman–Shapiro problem for the Bargmann–Fock space.

N. Nikolski. *Completeness of dilated functions. The case of binomials.*

After a short reminder on the cyclic function problem on the Hilbert multidisc (and its links to the Riemann Hypothesis), we give some new examples of cyclicity and noncyclicity (mostly for binomial functions).

S. Novikov. *Geometry of frames.*

Some problems of the Euclidean (unitary) spaces geometry will be discussed in connection with attempts to solve the discrete phase problem. Certain special classes of frames (equiangular, full spark) will be presented.

P. Paramonov. *Uniform harmonic approximation on plane compacts and logarithmic capacity.*

We will discuss a new individual criterion for the uniform harmonic aproximability of functions on plane compacts in terms of the classical logarithmic capacity.

V. Peller. *Multiple operator integrals and functions of triples of noncommuting self-adjoint operators.*

I am going to speak about recent results on multiple operator integrals with integrands in Haagerup(-like) tensor products of L^∞ spaces. I am also going to show that unlike in the case of pairs, there is no Lipschitz type estimates for functions of triples of noncommuting self-adjoint operators in any Schatten–von Neumann norms.

S. Platonov. *An analog of a Titchmarsh theorem for the Fourier transform on locally compact Vilenkin groups.*

We consider problems related to the Fourier transform on locally compact Vilenkin groups. We obtain sharp estimates for decreasing in the mean of an L^2 function Fourier transform. In the setting of the locally compact Vilenkin

groups, we study an analogue of one classical Titchmarsh theorem, which describes the Fourier transforms of the Lipschitz functions from L^2.

A. Poltoratski. *Toeplitz Order.*

As a part of recently developed Toeplitz approach to the area of Uncertainty Principle in Harmonic Analysis, a new ordering of the set of inner functions in the upper half-plane is introduced. I will discuss the main definitions of Toeplitz Order along with new results and open problems.

S. Popenov. *Interpolation in convex domains by sums of series of exponentials.* (Joint work with S. G. Merzlyakov.)

We study the problem of interpolation in a convex domain by sums of the exponential series. We find conditions in the terms of the limit directions at infinity of exponents in the series and of interpolation nodes. For specially located interpolation nodes, we obtain interpolation criteria; in fact, the interpolation problem becomes equivalent to the point-wise approximation.

N. Rastegaev. *On spectral asymptotics of the Neumann problem for the Sturm-Liouville equation with arithmetically self-similar singular weight.*

Spectral asymptotics of the Neumann problem for the Sturm-Liouville equation with an arithmetically self-similar singular weight is considered. Previous results by A. A. Vladimirov and I. A. Sheipak, and also by the speaker, rely on the spectral periodicity property, which imposes significant restrictions on the self-similarity parameters. A new method is introduced to estimate the eigenvalue counting function, which makes it possible to consider a much wider class of self-similar measures.

N. Rautian, V. Vlasov. *Spectral analysis of functional differential equations with unbounded operator coefficients.*

We study functional differential and integrodifferential equations with unbounded operator coefficients in Hilbert spaces. The principal part of the equations under consideration is an abstract hyperbolic equation perturbed by terms with delay and terms containing Volterra integral operators. We prove that the initial boundary value problems in weighted Sobolev spaces are well posed on the positive semi-axis for the specified equations. We also

investigate the spectra of operator-valued functions, which are symbols of these equations in the autonomous case. We study the spectral problems for operator-valued functions, which are symbols of Volterra integrodifferential equations with unbounded operator coefficients in Hilbert spaces. Operator models of such type have many applications in the linear viscoelasticity theory, homogenization theory, heat conduction theory in media with memory, etc.

1. N. A. Rautian and V. V. Vlasov, Properties of solutions of integro-differential equations arising in heat and mass transfer theory, Transactions of the American Mathematical Society **75** (2014), 185–204.

O. Reinov. *On symmetry of spectra of linear operators in Banach spaces.*

It was shown by M. I. Zelikin (2007) that the spectrum of a nuclear operator in a separable Hilbert space is central-symmetric if and only if the spectral traces of all odd powers of the operator are equal to zero. The criterium can not be extended to the case of general Banach spaces: It follows from Grothendieck–Enflo results that *there exists a nuclear operator U in the space l_1 such that* $\operatorname{trace} U = 1$ *and* $U^2 = 0$.

B. Mityagin (2016) has generalized Zelikin's criterium to the case of compact operators (in Banach spaces) for which certain powers are nuclear.

We give sharp generalizations of Zelikin's theorem (to the cases of subspaces of quotients of L_p-spaces) and of Mityagin's result (for the case where the operators are not necessarily compact). Our results are optimal: we present the following (sharp) generalization of Grothendieck–Enflo theorem.

Theorem. Let $p \in [1, \infty], p \neq 2$, $1/r = 1 + |1/2 - 1/p|$. There exists a nuclear operator V in l_p such that

(1) V is s-nuclear for each $s \in (r, 1]$;
(2) V is not r-nuclear;
(3) $\operatorname{trace} V = 1$ and $V^2 = 0$.

V. Riazanov. *On the boundary value problems in the plane. An alternative approach.*

The report is devoted to recent advances in non-classical solutions of the main boundary value problems such as the well-known Dirichlet, Hilbert,

Neumann, Poincare, and Riemann problems in the plane. Such solutions are essentially different from the variational solutions of the classical mathematical physics and are based on a nonstandard point of view of the geometrical function theory with a clear visual sense. The traditional approach of the latter is the meaning of the boundary values of functions at almost every boundary point in the sense of the so-called angular limits or limits along other prescribed classes of curves terminated at the boundary. This become necessary if we start to consider boundary data that are only measurable; and it is turned out to be useful under the study of problems in the field of mathematical physics, too. Thus, we essentially widen the notion of solutions and, furthermore, we obtain infinite-dimensional spaces of solutions for all boundary value problems mentioned above. The latter comment also concerns the Laplace equation as well as its counterparts in the potential theory for inhomogeneous and anisotropic media.

L. Slavin. *Dimension-free estimates for semigroup BMO and related classes.* (Joint work with P. Zatitiskiy.)

Let K_t be either the heat or the Poisson kernel on $\mathbb{R}^n$ and consider $\mathrm{BMO}_K(\mathbb{R}^n)$ equipped with the norm

$$\|\varphi\|_K := \sup_{z\in\mathbb{R}^{n+1}_+} \left(\varphi^2(z) - \varphi(z)^2\right)^{1/2},$$

where $g(z)$ denotes the K-extension of a function g on $\mathbb{R}^n$ into the upper half-space: $g(x,t) = (K_t * g)(x)$.

We establish the following transference principle between the classical space $\mathrm{BMO}(Q)$ on an interval and $\mathrm{BMO}_K(\mathbb{R}^n)$: If an integral functional admits an estimate on $\mathrm{BMO}(Q)$, then the same estimate holds for $\mathrm{BMO}_K(\mathbb{R}^n)$, with all Euclidean averages replaced by K-averages. In particular, all such estimates are dimension-free. The proof uses Bellman functions for $\mathrm{BMO}(Q)$ as locally concave majorants for their K-analogs, in conjunction with the probabilistic representation of the kernel K_t. Analogous results hold for related function classes, such as A_p.

S. Treil. *Clark model in general situation.*

I'll discuss recent developments in the theory of Clark model in the general case of non-inner characteristic function (not purely singular spectral measure). The case of rank one perturbations will be described in details; the recently developed theory for finite rank perturbations will be discussed.

A. Vasin. *A T(P) theorem in Campanato spaces on domains.*

Recently, X. Tolsa and M. Pratz have proved a T(P) theorem for a large family of Calderón-Zygmund operators in Sobolev spaces $W^{n,p}(D)$ on Lipschitz domains in $\mathbb{R}^d$. We investigate a different situation when one has to check the boundedness not only for the characteristic function, but for a finite collection of polynomials restricted to the domain under consideration. So, assuming that ω is a general growth function, we prove a T(P) theorem for the Campanato spaces $\mathcal{C}_\omega$ on Lipschitz domains in $\mathbb{R}^d$.

R. Zarouf. *Schäffer's conjecture, Fourier coefficients of Blaschke products and Jacobi polynomials with varying parameters.*

This work contains results that we found on our way to a deeper understanding of Schäffer's conjecture about inverse operators. Three main topics are covered in this work:

1. The first one is devoted to Schäffer's question: what is the best $\mathcal{S} = \mathcal{S}(n)$ such that

$$|\det T| \cdot \|T^{-1}\| \le \mathcal{S}\|T\|^{n-1}$$

holds for *any* induced matrix norm and *any* invertible complex $n \times n$ matrix. J. J. Schäffer proved that $\mathcal{S} \le \sqrt{en}$ and conjectured that $\mathcal{S}$ was bounded for any norm and choice of invertible matrices. Schäffer's conjecture was rebutted by Gluskin–Meyer–Pajor who determined an appropriate norm and a sequence of matrices $T = T(n)$ along which $\mathcal{S}$ grows unboundedly. While the choice of the norm turned out to be rather straightforward, the construction of suitable T led into deep number theory. In the currently most successful approach the eigenvalues of T should satisfy a Turàn-type power sum inequality. This method shows that $\mathcal{S} \ge \sqrt{n}(1 - \mathcal{O}(1/n))$ but it only proves the existence of T with growing $\mathcal{S}$. Finding the optimal $\mathcal{S}$ and the construction

of explicit sequences of matrices remain open tasks. Here we take a constructive approach to Schäffer's conjecture. We derive new upper estimates on $\mathcal{S}$ and we construct an explicit class of matrices that reaches the asymptotic growth due to Schäffer's theorem. Our framework naturally extends to provide sharp estimates on the resolvent $\|(\zeta - T)^{-1}\|$ when $|\zeta| \leq \|T\|$ and ζ does not intersect the spectrum of T.

2. A key ingredient in our approach will be to investigate l_p-norms of Fourier coefficients of a Blaschke product, which is an interesting and well-studied topic in its own right. When b is an elementary Blaschke factor it is known that in the parameter range $p \in [1, 2]$ we have

$$\|\hat{b}^n\|_{l_p} \leq C_p n^{\frac{2-p}{2p}}.$$

We show that this remains valid when $p \in [1, 4)$ but the corresponding asymptotic behavior is no longer the same for $p = 4$ and $p \in (4, +\infty]$.

3. Finally, on our way, we prove new estimates for the asymptotic behavior of Jacobi polynomials with varying parameters and we highlight some flaws in the established literature on this topic.

P. Zatitskii. *On a mysterious coincidence between two extremal problems.* (Joint work with A. A. Logunov, D. M. Stolyarov and V. I. Vasyunin.)

We will discuss two extremal problems, between the solutions of which we have found an unexpected coincidence. The first problem is sufficiently well studied and is related to estimates of integral functionals on the BMO space, while the second one is connected to estimates of the martingale transform on the L^∞ space.