MORE ABOUT UNCERTAINTY PRINCIPLE: THE LAST TALKS WITH V.P.

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ABSTRACT
I will tell about our discussion with Victor during last 2 years about the problem which I posed few years ago and which was extremely closely related to his interests and to uncertainty principle and very simple by its posing. The question was not to answer on the question — positive answer was done in terms of representation theory (of Heisenberg group) many years ago, — but to give a simple and direct proof. He had prepared two handwritten pages of letter to me(which I will show) with his own reformulation of my question. But the simple proof still does not exist.
(Remark after the talk: Such simple proof was suggested by professor B.Mityagin after the end of my talk).
But the goal of the presentation is to explain the connextion of the fact and similar ones with representation theory and its applications. The fact is the following: if a measurable function f on $\mathbb{R}$ has properties
1)$\forall t \in \mathbb{R} \quad f(t+\cdot) - f(\cdot) \in L^2_m(\mathbb{R})$,
2)$\forall s \in \mathbb{R} \quad \sin(s\cdot)f(\cdot) \in L^2_m(\mathbb{R})$
then $f \in L^2_m(\mathbb{R})$,

here m is Lebesgue measure on $\mathbb{R}$, the condition 2) is evidently equivalent the the condition 1) for Fourier transform $\tilde{f}$ of the function f.
This fact is one of reformulation of the uncertainty principle and the letter of VP which I presented contains some arguments.
From point of view of representation theory this fact is equivalent to the well-known property of the Heisenberg group *Heis* (the group of all upper triangle unipotent matrices of order 3 over the real numbers):

Theorem

the space of 1-cohomology $H^1(Heis, \pi)$ with values in any irreducible unitary (non-trivial) representations of the group Heis is trivial.

The group $H^1(G, \pi)$ of the first cohomology of the group G with coefficients in the unitary representation π in the Hilbert space H_π of the group G is the group of classes of equivalence of the cocycles; cjcyvle is a continuous map:

$$\beta : G \to H_\pi; \beta(g_1 \cdot g_2) = \beta(g_1) + \pi(g_1)\beta(g_2),$$

and equivalence of the cocycles means: $\beta_1 \sim \beta_2$ iff there exist $h \in H_\pi$ s.t.$\beta_1(g) - \beta_2(g) = \pi(g)h - h$.
In its turn this is a partial case of the general fact proved by several mathematicians that the same claim is true for all nilpotent Lie groups. I give the sketch of the proof of this fact which use some fact from representation theory. But Situation is much more interesting and open for for example for solvable Lie groups. F.e. for the group $Aff(2)$ of all upper triangle matrices of the order 2, the group H^1 is not trivial. The question which is under the consideration now is whether the space of cohomology $H^1(G)$ for so called Iwasawa subgroup (minimal Borel subgroup) of the semi-simple real Lie group is also nontrivial for some irreducible exact representation of (G)?