Euler International Mathematical Institute

24th St.Petersburg Summer Meeting in Mathematical Analysis and a Summer School for Young Scientists

June 25-30, 2015

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CONFERENCE PROGRAM

THURSDAY, June 25

09:30–10:30 REGISTRATION

10:30–11:20 **A. Marcus.** *A short course on finite free probability, Lecture 1.*

Coffee break

11:55–12:45 **V. Peller.** *Multiple operator integrals in perturbation theory, Lecture 1.*

Lunch

15:00–15:20 **Y. Belov.** *Regularity of canonical systems.*

15:25–15:45 **K. Dyakonov.** *Functions in Bloch-type spaces and their moduli.*

15:50–16:10 **L. Slavin.** *Inequalities for BMO on alpha-trees.*

Coffee break

16:35–16:55 **R. Zarouf.** *Maximum of the resolvent over matrices with given spectrum.*

17:00–17:20 **I. Videnskii.** *An analog of Blaschke product for Hilbert space with Nevanlinna-Pick kernel.*

17:30 WELCOME PARTY

FRIDAY, June 26

10:00–10:50 **A. Marcus.** *A short course on finite free probability, Lecture 2.*

Coffee break

11:15–12:05 **V. Peller.** *Multiple operator integrals in perturbation theory, Lecture 2.*

12:10–12:50 **A. Poltoratski.** *A characterization of Weyl inner functions for Schrödinger equations.*

Lunch

15:00–15:20 **G. Amosov.** *On the representation of the space of Schwartz operators and its dual on the plane.*

15:25–15:45 **S. Platonov.** *About spectral synthesis on element-wise Abelian groups.*

15:50–16:10 **A. Mirotin.** *On compact Hankel operators over compact Abelian groups.*

Coffee break

16:35–16:55 **A. Gaisin.** *Quantitative estimation of minimality of the exponential system* $\left(e^{\lambda_n z}\right)$ $(\lambda_n > 0)$ *in* $C[0,\delta]$.

17:00–17:20 **R. Gaisin.** *The Denjoy-Carleman theorem for Jordan domains.*

17:25–17:45 **S. Popenov.** *Interpolation by the series of exponentials in* $H(D)$ *from the real interpolation nodes.*

SATURDAY, June 27

10:00–10:50 **A. Marcus.** *A short course on finite free probability, Lecture 3.*

Coffee break

11:15–12:05 **V. Peller.** *Multiple operator integrals in perturbation theory, Lecture 3.*

12:10–12:50 **E. Korotyaev.** *Hardy Spaces and Schrödinger operators on lattices.*

Lunch

FREE AFTERNOON, EXCURSION

SUNDAY, June 28

FREE MORNING

14:00–14:40 **D. Yakubovich.** *Completeness of rank one perturbations of compact normal operators with lacunary spectrum.*

14:45–15:25 **M. Belishev.** *A functional model of a symmetric semi-bounded operator in inverse problems on manifolds.*

Coffee break

15:50–16:10 **E. Gluskin.** *How a rotated cube looks on a complex plane.*

16:15–16:35 **N. Shirokov.** *Smoothness of conformal mapping on a subset of boundary.*

16:40–17:00 **O. Reinov.** *On a question of Boris Mitjagin.*

18:00–19:00 CONCERT (PDMI, Fontanka, 27)

19:45 CONFERENCE DINNER

MONDAY, June 29

10:00–10:40 **S. Charpentier.** *Γ-supercyclicity.*

Coffee break

11:05–11:45 **K. Fedorovskiy.** *Carathéodory domains and Rudin's converse of the maximum modulus principle.*

11:50–12:30 **P. Gauthier.** *Density of polynomials in classes of functions on products of planar domains.*

Lunch

15.00–15.20 **H. Hato-Bommier.** *Products of Toeplitz operators and Sarason's conjecture on weighted Fock spaces.*

15:25–15:45 **I. Antipova.** *Fundamental correspondence for multidimensional Mellin transform of rational functions.*

15:50–16:10 **T. Sadykov.** *Analytic complexity of cluster trees.*

Coffee break

16:35–16:55 **N. Rautian, V. Vlasov.** *Spectral analysis of Volterra integro-differential equations arising in viscoelasticity theory.*

17:00–17:20 **N. Osipov.** *Two types of Rubio de Francia operators on Besov and Triebel-Lizorkin spaces.*

17:25–17:45 **P. Paramonov.** *Smooth approximations by solutions of second order elliptic equations.*

TUESDAY, June 30

10:00–10:40 **M. Rudelson.** *Delocalization of eigenvectors of random matrices with independent entries.*

Coffee break

11:05–11:45 **M. Malamud.** *Uniqueness results for systems of ODE and canonical systems.*

11:50–12:30 **H. Woracek.** *Perturbation of chains of de Branges spaces.*

PARTICIPANTS

ABAKUMOV, Evgeny Valerievich
University Paris-Est
5 Boulevard Descartes, Champs-sur-Marne, Marne-la-Vallée, 77454 France
Email: evgueni.abakoumov@u-pem.fr

ALEKSANDROV, Aleksei Borisovich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: alex@pdmi.ras.ru

AMOSOV, Grigori Gennadievich
Steklov Mathematical Institute
8, Gubkina, Moscow, 119991 Russia
Email: gramos@mi.ras.ru

ANTIPOVA, Irina Avgustovna
Siberian Federal Uneversity
79, Svobodny, Krasnoyarsk, 660041, Russia
Email: iantipova@sfu-kras.ru

BARANOV, Anton Dmitrievich
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: anton.d.baranov@gmail.com

BASOK, Mikhail Konstantinovich
Saint Petersburg State University and
St. Petersburg Department of V.A.Steklov Mathematical Institute
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: m.k.basok@gmail.com

BELISHEV, Mikhail Igorevich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: belishev@pdmi.ras.ru

BELOV, Yurii Sergeevich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: j_b_juri_belov@mail.ru

BESSONOV, Roman Viktorovich
Saint Petersburg State University and
St. Petersburg Department of V.A.Steklov Mathematical Institute
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: bessonov@pdmi.ras.ru

BOROVIK, Ekaterina
Bauman Moscow State Technical University
5, 2-nd Baumanskaya str., Moscow, 105005 Russia
Email: katrina_borovik@mail.ru

CHARPENTIER, Stéphane
Aix Marseille Université
39, rue F. Joliot Curie, Marseille, Cedex 13, 13453 France
Email: stephane.charpentier.amu@gmail.com

DOUBTSOV, Evgeniy Sergeevich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: dubtsov@pdmi.ras.ru

DUBASCHINSKY, Mikhail Borisovich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: mikhail.dubashinskiy@gmail.com

DYAKONOV, Konstantin Mikhailovich
ICREA and Universitat de Barcelona
Gran Via 585, E-08007 Barcelona, Spain
Email: konstantin.dyakonov@icrea.cat

EIDERMAN, Vladimir Yakovlevich
Indiana University
831 E 3rd st, Department of Mathematics, Bloomington IN 47405, USA
Email: veiderma@indiana.edu

FEDOROVSKIY, Konstantin Yurievich
Bauman Moscow State Technical University
5, 2-nd Baumanskaya str. Moscow, 105005 Russia
Email: kfedorovs@yandex.ru

GAISIN, Ahtyar Magazovich
Ufa Institute of Mathematics
112, Chernyshevsky str., Ufa, 450008 Russia
Email: gaisinam@mail.ru

GAISIN, Rashit Ahtyarovich
Bashkir State University
25/2, ap. 67, S.Perovskaya str., Ufa, 450092 Russia
Email: rashit.gajsin@mail.ru

GAMAL, Mariya Feliksovna
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: gamal@pdmi.ras.ru

GAUTHIER, Paul
Université de Montréal
Département de mathématiques et de statistique, Université de Montréal, CP6128,
Centreville Montréal, Québec H3Y 1Y8 Canada
Email: gauthier@dms.umontreal.ca

GLADKAYA, Anna Vladimirovna
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: anna.v.gladkaya@gmail.com

GLUSKIN, Efim Davydovich
Tel Aviv University
Ramat Aviv, Haim Levanon str., Tel Aviv, 69978 Israel
Email: gluskin@post.tau.ac.il

GORDON, Joseph
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: joseph-gordon@yandex.ru

GURYANOVA, Irina Edwardovna
Financial University under the Government of the Russian Federation
49b, Lenigradskiy pr., Moscow, 125993 Russia
Email: irinagur@list.ru

HATO-BOMMIER, Helene
Aix-Marseille University, France
39, rue F. Joliot Curie, Marseille Cedex 13 13453 France
Email: helene.bommier@gmail.com

HAVIN, Viktor Petrovich
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: havin@vh1621.spb.edu

IVANOV, Sergei Alekseevich
St.Petersburg Branch of Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation
1, Mendeleev line, St.-Petersburg, 199034 Russia
Email: sergei.a.ivanov@mail.ru

KALITA, Eugene Alexandrovich
Institute of Applied Mathematics and Mechanics
74, Roza Luxemburg st., Donetsk, 83114 Ukraine
Email: ekalita@mail.ru

KALTENBÄCK, Michael
Vienna University of Technology
8-10 Wiedner Haupstrasse, 1040 Wien, Austria
Email: michael.kaltenbaeck@tuwien.ac.at

KAPUSTIN, Vladimir Vladimirovich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: kapustin@pdmi.ras.ru

KISLYAKOV, Sergey Vitalievich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: skis@pdmi.ras.ru

KONONOVA, Anna Aleksandrovna
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: anya.kononova@gmail.com

KOROTYAEV, Evgeny Leonidovich
Saint Petersburg State University
1, Ulyanovskaya, St.Petersburg, 198504 Russia
Email: korotyaev@gmail.com

KOTOCHIGOV, Alexandr Mihailovich
Saint Petersburg State Electrotechnical Universite
5, Popova, St. Petersburg 197367, Russia
Email: amkotochigov@gmail.com

LISHANSKII, Andrei Alexandrovich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: lishanskiyaa@gmail.com

LOGUNOV, Alexander Andreevich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: log239@yandex.ru

MALAMUD, Mark Mikhailovich
Institute of Applied Mathematics and Mechanics
74, Roza Luxemburg st., Donetsk, 83114 Ukraine
Email: mmm@telenet.dn.ua

MARCUS, Adam
Yale University, and Crisply, INC
7 Research Dr. / Woodbridge, CT 06525
Email: adam.marcus@yale.edu

MATVEENKO, Sergey Georgievich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: Matveis239@gmail.com

MIROTIN, Adolf Ruvimovich
F. Skorina Gomel State University
104, Sovetskaya, Gomel, Belarus 246019
Email: amirotin@yandex.ru

MUKOSEEVA, Ekaterina Vadimovna
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: emukoseeva@gmail.com

NAZAROV, Alexander Ilyich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: al.il.nazarov@gmail.com

NIKITENKOVA, Svetlana Pavlovna
Lobachevsky State University of Nizhny Novgorod
23 korp 4, Gagarina, Nizhny Novgorod, 603950 Russia
Email: spnikitenkova@yandex.ru

NIKOLSKI, Nikolai Kapitonovich
Université de Bordeaux 1
351, Cours de la Liberation, Talence, 33405 France
Email: nikolski@math.u-bordeaux1.fr

OSIPOV, Nikolai Nikolaevich
St. Petersburg Department of V.A.Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: nicknick@pdmi.ras.ru

PARAMONOV, Petr Vladimirovich
Moscow State University
GSP-1, Leninskie Gory, Moscow 119991 Russia
Email: petr.paramonov@list.ru

PELLER, Vladimir Vsevolodovich
Michigan State University
East Lansing, MI 48864, USA
Email: peller@math.msu.edu

PLATONOV, Sergey Sergeevich
Petrozavodsk State University
33, Lenina, Petrozavodsk 185910 Russia
Email: ssplatonov@yandex.ru

POLTORATSKI, Alexei Gennadievich
Texas A&M University
College Station, TX 77843, USA
Email: apoltora@gmail.com

POPENOV, Sergey Victorovich
Ufa Institute of Mathematics
112, Chernyshevsky str., Ufa, 450008 Russia
Email: spopenov@gmail.com

PRUCKNER, Raphael
Vienna University of Technology
8-10 Wiedner Haupstrasse, 1040 Wien, Austria
Email: raphael.pruckner@tuwien.ac.at

PYSHKIN, Alexey
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: aapyshkin@gmail.com

RAUTIAN, Nadezhda Aleksandrovna
Moscow State University
GSP-1, Leninskie Gory, Moscow 119991 Russia
Email: nrautian@mail.ru

REINOV, Oleg Ivanovich
Saint Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: orein51@mail.ru

REVYAKOV, Mikhail Ilyich
St. Petersburg Math. Soc.
Email: revyakov.m@gmail.com

ROMANOV, Roman Vladimirovich
Saint Petersburg State University
1, Ulyanovskaya, St.Petersburg, 198504 Russia
Email: morovom@gmail.com

ROTKEVICH, Aleksandr Sergeevich
Saint Petersburg State University
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: rotkevichas@gmail.com

RUDELSON, Mark
University of Michigan
530 Church St., Ann Arbor, MI 48109, USA
Email: rudelson@umich.edu

SADYKOV, Timur Mradovich
Russian Plekhanov University
36, Stremyannyj, Moscow, 117997 Russia
Email: sadykov1976@mail.ru

SHIROKOV, Nikolai Alexeevich
Saint Petersburg State University and Higher School of Economics
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: nikolai.shirokov@gmail.com

SLAVIN, Leonid
University of Cincinnati
University of Cincinnati, PO Box 210025, Cincinnati, Ohio 45221-0025
Email: leonid.slavin@uc.edu

VASIN, Andrei Vasilievich
State University of Maritime & Inland Shipping
5/7, Dvinskaya, St-Petersburg, 198035 Russia
Email: andrejvasin@gmail.com

VASYUNIN, Vasily Ivanovich
St. Petersburg Department of V.A. Steklov Mathematical Institute
27, Fontanka, St. Petersburg, 191023 Russia
Email: vasyunin@pdmi.ras.ru

VIDENSKII, Ilya Victorovich
St.Petersburg State University
28, Universitetskii pr., Petrodvorets, St. Petersburg, 198504 Russia
Email: ilya.viden@gmail.com

VLASOV, Victor Valentinovich
Moscow State University
GSP-1, Leninskie Gory, Moscow 119991 Russia
Email: vikvvlasov@rambler.ru

WORACEK, Harald
Vienna University of Technology
8-10 Wiedner Haupstrasse, 1040 Wien, Austria
Email: harald.woracek@tuwien.ac.at

YAKUBOVICH, Dmitry Vladimirovich
Universidad Autonoma de Madrid, and
Instituto de Ciencias Matemáticas (CSIC - UAM - UC3M - UCM)
Cantoblanco 28049, Spain
Email: dmitry.yakubovich@uam.es

ZAROUF, Rachid
Aix Marseille Université
39, rue F. Joliot Curie, Marseille Cedex 13 13453 France
Email: rzarouf@gmail.com

ZATITSKIY, Pavel Borisovich
Saint Petersburg State University and
St. Petersburg Department of V.A.Steklov Mathematical Institute
29B, 14th Line Vasilyevsky Island, St.Petersburg, 199178 Russia
Email: paxa239@yandex.ru

ABSTRACTS OF COURSES

A. Marcus. *A short course on finite free probability.*

The theory of free probability has been a useful tool in a number of different areas of mathematics. Most notable, perhaps, is its use in understanding the asymptotic properties of random matrices. One of the limitations of free probability, however, is that the true power of free independence can only be realized in infinite dimensional situations. The goal of this short course will be to introduce a new theory of "finite free probability," which was developed as a way to address these shortcomings.

In the first talk, I will introduce the main concepts of noncommutative probability that I will need and show how they can be adapted to a finite setting. In particular, I hope to show how the finite free additive convolution is a natural extension of the free additive convolution. In the process we will see some of the advantages of having finite structures, including an interesting relationship between classical independence and finite free independence that does not seem to show up in the infinite case.

In the second talk, I will discuss some of the theoretical properties of the finite free additive convolution, including a finite free central limit theorem and finite free entropy. In particular, we will see more of the advantages of dealing with finite structures, including a number of recent and powerful results in the theory of polynomials. This leads to an inequality that suggests that one can actually do better in finite settings than the asymptotic limit would imply.

In the third talk, I will give an application of this new theory to the problem of showing the existence of graphs with particular spectral properties. This will include an argument that we call "the method of interlacing polynomials" which can be seen as an analogue of the probabilistic method from classical probability. In short, this will use more of the recent and powerful results in polynomial theory to show the existence of structures which behave at least as well as the asymptotic limit. I also hope to pose some open questions and suggest possible directions for future research.

The talks will be based primarily on work that was done in collaboration with Dan Spielman and Nikhil Srivastava, but will include some work with Dima Shlyakhtenko.

V. Peller. *Multiple operator integrals in perturbation theory.*

Double operator integrals appeared in a paper by Yu. L. Daletskii and S. G. Krein who observed that such integrals arise in a natural way in perturbation theory. Later M. S. Birman and M. Z. Solomyak developed a beautiful theory of double operator integrals. Double operator integrals are expressions of the form

$$\iint \Phi(x_1, x_1)\, dE_1(x_1) T\, dE_2(x_2),$$

where E_1 and E_2 are spectral measures, T is a bounded operator on Hilbert space and Φ is a measurable function satisfying certain assumptions. Functions satisfying such assumptions are called *Schur multipliers.*

Birman and Solomyak observed that if f is a function on $\mathbb{R}$ such that the divided difference $\mathfrak{D}f$, $(\mathfrak{D}f)(s,t) \overset{\text{def}}{=} (f(s) - s(t))(s-t)^{-1}$, is a Schur multiplier, then for arbitrary self-adjoint operators A and B with bounded $A - B$, the following formula holds:

$$f(A) - f(B) = \iint (\mathfrak{D}f)(s,t)\, dE_A(s)(A - B)\, dE_B(t),$$

where E_A and E_B are the spectral measures of A and B. This implies that

$$\|f(A) - f(B)\| \le \text{const}\, \|A - B\|.$$

Functions satisfying this inequality are called *operator Lipschitz*. It turns out that the converse is also true: if f is operator Lipschitz, then $\mathfrak{D}f$ is a Schur multiplier.

I am going to give sufficient conditions and necessary conditions for operator Lipschitzness.

Then I will introduce the notion of operator Hölder functions of order α, $0 < \alpha < 1$. It turns out that unlike in the case of operator Lipschitz functions, the class of operator Hölder functions of order α coincides with the class of Hölder functions of order α.

I am also going to consider the problem of estimating the norms of $f(A) - f(B)$ in Schatten–von Neumann classes.

The situation for functions of normal operators and for functions of n-tuples of commuting self-adjoint operators is more complicated. However, it turns out that the results mentioned above can be generalized.

In the final lecture I am going to speak about my recent joint results with A.B. Aleksandrov and F.L. Nazarov. I will introduce functions $f(A, B)$ of noncommuting self-adjoint operators A and B. This problem leads naturally to triple operator integrals:

$$\begin{aligned}
&f(A_1, B_1) - f(A_2, B_2) = \\
&= \iiint \big(\mathfrak{D}^{[1]}f\big)(x_1, x_2, y)\, dE_{A_1}(x_1)(A_1 - A_2)\, dE_{A_2}(x_2)\, dE_{B_1}(y) \\
&+ \iiint \big(\mathfrak{D}^{[2]}f\big)(x, y_1, y_2)\, dE_{A_2}(x)\, dE_{B_1}(y_1)(B_1 - B_2)\, dE_{B_2}(y_2),
\end{aligned}$$

where the divided differences $\mathfrak{D}^{[1]}f$ and $\mathfrak{D}^{[2]}f$ are defined by

$$\mathfrak{D}^{[1]}f(x_1, x_2, y) \stackrel{\text{def}}{=} \frac{f(x_1, y) - f(x_2, y)}{x_1 - x_2},$$

and

$$\mathfrak{D}^{[2]}f(x, y_1, y_2) \stackrel{\text{def}}{=} \frac{f(x, y_1) - f(x, y_2)}{y_1 - y_2}.$$

To justify this formula we define Haagerup-like tensor products of the first kind and of the second kind and define triple operator integral for functions in such Haagerup-like tensor products. Then we prove that if f is a function on $\mathbb{R}^2$ of Besov class $B^1_{\infty,1}(\mathbb{R}^2)$, then $\mathfrak{D}^{[1]}f$ belongs to the Haagerup tensor product of the first kind, while $\mathfrak{D}^{[2]}f$ belongs to the Haagerup tensor product of the second kind. This implies that if $1 < p < 2$, $f \in B^1_{\infty,1}(\mathbb{R}^2)$, then the following Lipschitz type estimate in the Schatten–von Neumann norm $\boldsymbol{S}_p$ holds:

$$\|f(A_1, B_1) - f(A_2, B_2)\|_{\boldsymbol{S}_p} \le \operatorname{const} \max\{\|A_1 - A_2\|_{\boldsymbol{S}_p}, \|A_1 - A_2\|_{\boldsymbol{S}_p}\}.$$

On the other hand, it turns out that there is no such a Lipschitz estimate in $\boldsymbol{S}_p$ for $p > 2$ as well as in the operator norm.

ABSTRACTS OF TALKS

G. Amosov. *On the representation of the space of Schwartz operators and its dual on the plane.*

A linear bounded operator T in the Hilbert space $H = L^2(\mathbb{R})$ is said to be a Schwartz operator if it is bounded with respect to a family of seminorms $\|T\|_{f,n,m} = \|M^n D^m T f\|_H$, where $D = \frac{d}{dx}$, M is the operator of multiplication by x, $n, m = 0, 1, 2, \ldots$, and f belongs to the Schwartz space $S(\mathbb{R})$. The definition (M. Keyl, J. Kiukas, R.F. Werner, 2015) is equivalent to the claim that T is an integral operator with the kernel $\rho(\cdot,\cdot) \in S(\mathbb{R}^2)$. The linear space $\mathcal{T}$ of all Schwartz operators is a Frechet space. The transformation

$$\omega(x,\varphi) = \frac{1}{2\pi} \int\limits_{\mathbb{R}^2} e^{i(y\cos\varphi - x)t} \rho\left(y + \frac{t\sin\varphi}{2}, y - \frac{t\sin\varphi}{2}\right) dydt$$

results in the function $\omega(\cdot,\varphi) \in S(\mathbb{R})$ for any fixed $\varphi \in [0, 2\pi]$ and $\omega(x,\cdot) \in C^\infty$. We describe the structure of the dual $\mathcal{T}'$ for the Frechet space $\mathcal{T}$ in the representation associated to the map $T \to \omega(x,\varphi)$. It is shown that there exists a map $\mathcal{T}' \ni A \to a(x,\varphi)$ such that

$$\int\limits_0^{2\pi} \int\limits_{\mathbb{R}} \omega(x,\varphi) a(x,\varphi) dx d\varphi = Tr(TA)$$

if A is a polynomial in the unbounded operators M and D.

I. Antipova. *Fundamental correspondence for multidimensional Mellin transform of rational functions.* (A joint work with A. Tsikh and A. Schuplev.)

An arbitrary pair of convex domains $\Theta, U \subset \mathbb{R}^n$ codes the isomorphic functional spaces M_Θ^U, W_U^Θ transforming to each other by direct and inverse Mellin transforms (Antipova, 2007). It means that

Θ and U define domains in which functions of the spaces M_{Θ}^{U} and W_{U}^{Θ} are holomorphic. Moreover, Θ and U predetermine the asymptotics of functions. The subspace of rational functions in the space M_{Θ}^{U} has simple identification in the case of quasielliptic denominators. It is known that such functions do not vanish in a suitable toric compactification of the space $\mathbb{R}^n$ (Yermolaeva-Tsikh, 1996). A quasielliptic polynomial admits power minorant x^u , $u \in U$, where U is its Newton polytope. Nilsson and Passare (2013) proved that the Mellin transform of a rational function in n variables with quasielliptic denominator admits a representation in the form of nonconfluent product of gamma-functions $\Gamma(\langle \alpha, s \rangle + a_\alpha)$, where α belongs to the set of normals of the denominator Newton polytope, and an entire function of exponential type. In my talk, I will present the improved Nilsson-Passare representation for the direct Mellin transform which can be treated as the fundamental correspondence for it. The proof is based on a representation of the integration orthant $\mathbb{R}^n_+$ in the form of an $(n-1)$-dimensional family of one-parameter curves, and the Leray multidimensional residue theory.

M. Belishev. *A functional model of a symmetric semi-bounded operator in inverse problems on manifolds.*

We deal with the problem of reconstruction of a Riemannian manifold Ω from its boundary (dynamical and/or spectral) inverse data. It is shown that the problem can be solved by constructing a functional model of a symmetric semi-bounded operator L_0 determined by the inverse data. A basic element of the construction is the so-called *wave spectrum* Ω_{L_0} of L_0, which is introduced via the trajectories of a dynamical system governed by the wave equation $u_{tt} + L_0^* u = 0$. The spectrum Ω_{L_0} is endowed with relevant topology and metric, which turns it to a Riemannian manifold $\tilde{\Omega}$ such that $\tilde{\Omega} \overset{\text{isom}}{=} \Omega$ holds. Thus, $\tilde{\Omega}$ provides the solution to the reconstruction problem.

[1] M.I.Belishev, *Recent progress in the boundary control method.* Inverse Problems **23** (2007), no. 5, R1–R67.

[2] M.I.Belishev, *A unitary invariant of a semi-bounded operator in reconstruction of manifolds.* Journal of Operator Theory **69** (2013), no. 2, 299–326.

[3] M.I.Belishev and M.N.Demchenko, *Elements of noncommutative geometry in inverse problems on manifolds.* Journal of Geometry and Physics **78** (2014), 29–47.

Y. Belov. *Regularity of canonical systems.*

Let $Y(t,z)$ be a unique solution of the system

$$\frac{\partial}{\partial t}Y(t,z)J = zY(t,z)H(t),\ t\in[0,L],\ \ Y(0,z)=Id,\ \ J:=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}.$$

H is a Hamiltonian ($H\geq 0$, $trH\equiv 1$). Put $(A_t(z),B_t(z)) := (1,0)Y(t,z)$, $t\in[0,L]$, $E_t(z):=A_t(z)-iB_t(z)$. Then E is an entire function of Hermite-Biehler class. De Branges space $\mathcal{H}(E)$ corresponds to the Hamiltonian H. We will say that μ is a spectral measure for $\mathcal{H}(E)$ if $\mathcal{H}(E)$ is isometrically embedded into $L^2(\mu)$.

One of the central results in the de Branges theory says that *any* measure which is summable with respect to the weight $(1+x^2)^{-1}$ generates a chain of de Branges spaces (canonical systems) isometrically embedded into $L^2(\mu)$. In addition, this chain is dense in $L^2(\mu)$.

One of the classical questions is to find the exponential type of the chain (of de Branges subspaces). This famous question has been studied for many years by different mathematicians (Krein, Mergelyan, de Branges, Koosis, Poltoratski).

We will study a similar question related to the regularity of de Branges chains. Under certain regularity conditions on the weight w (these conditions imply that the exponential type of the measure $w(x)dx$ is infinite), we will show that the chain has a regular growth.

More precisely, if two de Branges spaces from the chain have the same exponential type, then these spaces coincide.

S. Charpentier. Γ-*supercyclicity.* (A joint work with R. Ernst and Q. Menet.)

A linear operator T on a Banach space X is called hypercyclic if there exists a vector x in X such that the orbit $Orb(x,T)$ of x under the action of T is dense in X. A strictly weaker notion is that of supercyclicity: T is called supercyclic if there exists x in X such that the projective orbit $Orb(\mathbb{C}x,T)$ of x under T is dense in X. For a subset Γ of $\mathbb{C}$, we introduce the notion of Γ-supercyclicity, telling that T is Γ-supercyclic if there exists an x in X such that the set

$$Orb(\Gamma x,T) := \{\gamma T^n x,\, \gamma \in \Gamma,\, n \geq 0\}$$

is dense in X. In 2004, Leon and Möller proved that the unit circle $\mathbb{T}$ satisfies the following property: for any linear operator T on any Banach space X, T is $\mathbb{T}$-supercyclic if and only if T is hypercyclic. In this talk we will discuss the problem of describing the subsets of $\mathbb{C}$ which also enjoy this property.

K. Dyakonov. *Functions in Bloch-type spaces and their moduli.*

We give a new characterization of certain Lipschitz and Bloch type spaces on the ball in terms of the functions' moduli.

K. Fedorovskiy. *Carathéodory domains and Rudin's converse of the maximum modulus principle.*

We will discuss extensions of the converse to the maximum modulus principle. Namely, we will extend a classical Rudin's theorem from the unit disk to Carathéodory domains. Our approach is based on recently obtained properties of the conformal mappings of Carathéodory domains.

A. Gaisin. *Quantitative estimation of minimality of the exponential system* $\left(e^{\lambda_n z}\right)$ $(\lambda_n > 0)$ *in* $C[0,\delta]$.

For the exponential system $E = \left(e^{\lambda_n z}\right)$ with exponents satisfying Fejér condition or Levinson condition, we obtain an asymptotic estimation of the distance in $C[0,\delta]$ from the fixed exponential to the closure of the span of the system rest elements as $\delta \to 0$. The corresponding problem for arcs was posed in [1] by Korevaar (see p.216, problem 3).

[1] J. Korevaar, *Müntz approximation on arcs and Macintyre exponents.* Lecture notes in Mathematics. Complex analysis. Joensuu **747** (1978), 205–218.

R. Gaisin. *The Denjoy-Carleman theorem for Jordan domains.*

Let D be a Jordan domain. It is well known that the criteria of quasianalyticity of Carleman classes $H(D, M_n)$ with $M_n > 0$ for an angle and a disk can be paraphrased in terms of the Denjoy–Carleman theorem. We generalize these results to some Jordan domains of more general type. Also, we prove the following universal criterion: for any weakly uniform domain, the regular Carleman class $H(D, M_n)$ is quasianalytic if and only if the numbers M_n satisfy the Mandelbrojt–Bang condition.

P. Gauthier. *Density of polynomials in classes of functions on products of planar domains.* (A joint work with V. Nestoridis.)

Let I be a set of indices of cardinality $|I|$ and let X be a product of plane domains indexed by I. Let f be a complex-valued function, defined on X. We consider the problem of approximating f by polynomials of $|I|$ variables. In this work, we allow the cardinality $|I|$ to be arbitrary.

E. Gluskin. *How a rotated cube looks on a complex plane.* (A joint work with Y. Ostrover.)

The symplectic capacity problem inspires the question if for any (or at least for random) rotation $O(2n)$ there exists a complex plane L such that the area of the orthogonal projection on L of the image of the standard cube is bounded by some universal constant. We give the negative answer to this question.

I. Guryanova. *Volterra integral equations.*

We consider the equation

$$x(t) = f(t) + \int_{M(t)} K(t, s, x(s)) d\mu(s), \tag{1}$$

where $t \in \Omega$, Ω is a connected locally compact metric space; the measure μ is defined on the Borel sets $A \subset \Omega$; the functions $f : \Omega \to \mathbb{R}$ and $K : \Omega^2 \times \mathbb{R} \to \mathbb{R}$ are continuous; the image $M : \Omega \to 2^\Omega$ satisfies the following system of axioms, which generalize the notion of the Volterra type integral equation:

1. For any $t \in \Omega$, $M(t)$ is a compact set; moreover, for any open $\mathcal{J} \subset \Omega$, if $\mathcal{J} \cap M(t) \neq \emptyset$, then $\mu(\mathcal{J} \cap M(t)) \neq \emptyset$.
2. For any $t \in \Omega$, $\lim_{s \to t} \mu(M(t) \Delta M(s)) = 0$, where Δ denotes the symmetrical difference.
3. For any $t \in \Omega$, $s \in M(t) \Longrightarrow M(s) \subset M(t)$.
4. For any $t \in \Omega$, there exists $s \in M(t)$ such that $M(s) = \emptyset$.

We prove that there exists a solution of equation (1). The solution can be constructed by the method of consequent approximations.

H. Hato-Bommier. *Products of Toeplitz operators and Sarason's conjecture on weighted Fock spaces.* (A joint work with E. H. Youssfi and K. Zhu.)

The setting of this talk is the weighted Fock space F_m^2 of entire functions on $\mathbb{C}$ which are square integrable with respect to the measure $d\mu_m(z) = e^{-|z|^{2m}} dz$, $m > 0$, where dz is the normalized Lebesgue measure. In the context of the Segal–Bargmann space ($m = 1$), Cho, Park and Zhu studied the boundedness of the product of Toeplitz operators $T_u T_{\overline{v}}$ on F_1^2. We extend their work to the case of general $m \geq 1$, and give necessary and sufficient conditions on u, v in F_m^2 for the product $T_u T_{\overline{v}}$ to be bounded on F_m^2. In particular, we relate the boundedness of $T_u T_{\overline{v}}$ with the boundedness of product of the Berezin transforms of $|u|^2$ and $|v|^2$ (Sarason's conjecture).

E. Korotyaev. *Hardy Spaces and Schrödinger operators on lattices.* (A joint work with A. Laptev.)

We consider Schrödinger operators with complex potentials on multidimensional lattices. We assume that the potential is decaying sufficiently fast. We describe the spectral properties of the Schrödinger operator. In particular, we obtain trace formulas and some estimates of eigenvalues in terms of the potential. Our proof is based on results about the Hardy spaces in the disk.

M. Malamud. *Uniqueness results for systems of ODE and canonical systems.*

Let B be a non-singular diagonal $n \times n$ matrix and let $Q \in L^1[0,1] \otimes \mathbb{C}^{n\times n}$ be a potential matrix. Consider the following system of differential equations

$$(2) \qquad -iB^{-1}y' + Q(x)y = \lambda y, \quad y = \operatorname{col}(y_1, \ldots, y_n), \quad x \in [0,1].$$

Let $T \in \mathbb{C}^{n\times n}$ and $\det T \neq 0$. Denote by $W(\cdot, \lambda)$ the $n \times n$ fundamental matrix solution of equation (2) satisfying the initial condition $W(0, \lambda) = T$, $\lambda \in \mathbb{C}$.

The matrix function $W(\lambda) := W(1, \lambda)$ is called the monodromy matrix of the system (2) on the interval $[0, 1]$. Different results on the unique determination of a potential matrix Q by a certain part of the monodromy matrix will be discussed.

Similar uniqueness problems will be also discussed for canonical systems with non-singular Hamiltonian. We complete the results of Z. Leibenzon [1] and the author [2] on systems of ODE, as well as the results of L. De Branges [3] and D. Arov, H. Dym [4] regarding Hamiltonian systems.

[1] Z.L. Leibenzon, *The Connection between the Inverse Problem and the Completeness of the Eigenfunctions,* Doklady Acad. Sci. USSR **145** (1962), 519–522.

[2] M.M. Malamud, *Questions about the uniqueness in inverse problems for systems of differential equations on a finite interval*, Trans. Moscow Math. Soc. **60** (1999), 199–258.

[3] L. de Branges, *Some Hilbert spaces of entire functions. IV.*, Trans. Amer. Math. Soc. **105** (1962), 43–83.

[4] D.Z. Arov, H. Dym, *Bitangential direct and inverse problems for systems of integral and differential equations.* Encyclopedia of Math. and its Appl. **145**, Cambridge, 2012.

A. Mirotin. *On compact Hankel operators over compact Abelian groups.*

Let G be a compact Abelian group with totally ordered dual. The talk will contain generalizations of Hartman, Kroneker, Adamyan-Arov-Krein and Peller theorems for Hankel operators on the Hardy space $H^2(G)$. In particular, we have the following theorem:

Theorem. *A nontrivial finite rank Hankel operator on $H^2(G)$ exists if and only if the dual of G contains the first positive element χ_1. In this case, Hankel operator H_φ with symbol $\varphi \in H^\infty(G)$ is of finite rank if and only if the co-analytic part of φ has the form $R \circ \chi_1$, where R is a rational function with all its poles in the open unit disc. Moreover* $\operatorname{rank} H_\varphi = \deg R$.

N. Osipov. *Two types of Rubio de Francia operators on Besov and Triebel-Lizorkin spaces.* (A joint work with E. Malinnikova.)

Consider a finite or countable collection $\{\Delta_m\}$ of pairwise disjoint intervals in $\mathbb{R}$. Let $\{\varphi_m\}$ be a collection of Schwartz functions that are constructed from a single function φ by shifts and dilations of its Fourier transform so that $\operatorname{supp} \widehat{\varphi}_m = \Delta_m$. Consider two operators that transform scalar-valued functions to collections of functions by the formulas

$$Sf = \{f * \varphi_m\} \quad \text{and} \quad \widetilde{S} f = \left\{e^{-2\pi i\, a_m x}(f * \varphi_m)\right\},$$

where a_m are the left ends of Δ_m.

In 1983, Rubio de Francia proved that

$$\big\|\widetilde{S} f\big\|_{L^p(l^2)} \le C_{p,\varphi} \|f\| \quad \text{for} \quad 2 \le p < \infty,$$

where $C_{p,\varphi}$ does not depend on f or $\{\Delta_m\}$. The exponents $e^{-2\pi i\, a_m x}$ played a significant role in the proof, but since the L^p-norms are rotation-invariant, these exponents can be dropped, and so the operator S is also bounded on the same spaces. Now we note that most of the Besov spaces $\dot{B}^s_{pq}$ as well as Triebel–Lizorkin spaces $\dot{F}^s_{pq}$ are not rotation-invariant. An interesting phenomenon is that only one of our operators is bounded on some of such spaces and both of them are bounded on some other ones.

P. Paramonov. *Smooth approximations by solutions of second order elliptic equations.*

We will formulate some new criteria for the approximability of functions in individual form.

S. Platonov. *About spectral synthesis on element-wise Abelian groups.*

Let G be a locally compact Abelian group, $C(G)$ be the space of all complex-valued continuous functions on G. A closed linear subspace $\mathcal{H} \subseteq C(G)$ is said to be an invariant subspace if it is invariant with respect to the translations $\tau_y : f(x) \mapsto f(xy)$, $y \in G$. By definition, an invariant subspace $\mathcal{H} \subseteq C(G)$ admits strong spectral synthesis if $\mathcal{H}$ coincides with the closed linear span of the characters of G belonging to $\mathcal{H}$. We say that the strong spectral synthesis holds on a group G in the space $C(G)$ if every invariant subspace $\mathcal{H} \subseteq C(G)$ admits strong spectral synthesis. An element x of a topological group G is said to be compact if the smallest closed subgroup of G, which contains x, is compact. A topological group G is called an element-wise compact group if any element of G is compact. It can be proved that a locally compact Abelian group G is element-wise compact if and only if the dual group $\hat{G}$ is a zero-dimensional Abelian group.

Theorem. *The strong spectral synthesis holds on a locally compact Abelian group G in the space $C(G)$ if and only if G is element-wise compact.*

A. Poltoratski. *A characterization of Weyl inner functions for Schrödinger equations.*

This talk is devoted to applications of complex function theory to spectral problems for second order differential operators. We give a characterization of Weyl inner functions corresponding to Schrödinger equations on an interval. We apply this characterization to obtain new results in the so-called mixed spectral problems for Schrödinger operators.

S. Popenov. *Interpolation by the series of exponentials in $H(D)$ from the real interpolation nodes.* (A joint work with S.G. Merzlyakov.)

Let D be a convex domain in the complex plane that contains the set $\{\mu_k\}$ of real nodes which is discrete in D. We completely solve the interpolation problem with data on the set of real nodes by series of exponentials in $H(D)$. Criteria of interpolation are obtained for all possible types of domains as well as for all locations of limit points of $\{\mu_k\}$. Criteria are formulated in the terms of the boundary structure near the finite limit points and the location of limit set at infinity of exponents of interpolation series $\sum c_k \exp(\lambda_n z)$ which is convergent in $H(D)$. Furthermore, we solve a problem of interpolation by elements of invariant subspaces as well as by the series of functions, which are close to the exponential ones.

N. Rautian, V. Vlasov *Spectral analysis of Volterra integro-differential equations arising in viscoelasticity theory.*

We study integro-differential equations with unbounded operator coefficients in Hilbert spaces. The principal part of the equation is an abstract hyperbolic equation perturbed by summands with Volterra integral operators. These equations represent an abstract form of the integro-partial differential equations arising in viscoelasticity theory, heat conduction theory in media with memory, etc. A spectral analysis of the operator-valued functions, which are the symbols of considered integro-differential equations, is provided.

O. Reinov. *On a question of Boris Mitjagin.*

Boris Mitjagin asked me the following question at the Aleksander Pelczynski Memorial Conference (Bedlewo, 2015): Is it true that a product of two nuclear operators in Banach spaces can be factored through a trace class operator in a Hilbert space? We show that the answer is negative and we discuss related problems.

M. Rudelson. *Delocalization of eigenvectors of random matrices with independent entries.* (A joint work with R. Vershynin.)

Let A be an n by n random matrix with independent centered entries having exponential type tail decay and unit variances. We prove that, with high probability, the eigenvalues of A are delocalized, i.e. all coordinates of any unit eigenvector have the magnitude $O(n^{-1/2})$ up to logarithmic terms.

T. Sadykov. *Analytic complexity of cluster trees.*

The Kolmogorov-Arnold theorem yields a representation of a multivariate continuous function in terms of a composition of functions which depend on at most two variables. In the analytic case, understanding the complexity of such a representation naturally leads to the notion of the analytic complexity of (a germ of) a bivariate multi-valued analytic function. According to Beloshapka's local definition, the order of complexity of any univariate function is equal to zero while the n-th complexity class is defined recursively to consist of functions of the form $a(b(x, y) + c(x, y))$, where a is a univariate analytic function and b and c belong to the $(n - 1)$-th complexity class. Such a representation is meant to be valid for suitable germs of multi-valued holomorphic functions.

A randomly chosen bivariate analytic function will most likely have infinite analytic complexity. However, for a number of important families of special functions of mathematical physics their complexity is finite and can be computed or estimated. Using this, we introduce the notion of the analytic complexity of a binary tree, in particular, a cluster tree, and investigate its properties.

N. Shirokov. *Smoothness of conformal mapping on a subset of boundary.*

The aim of the talk is to state that the rate of smoothness of a conformal mapping is stable on a big subset of a boundary after perturbation on a set of arcs.

L. Slavin. *Inequalities for BMO on alpha-trees.* (A joint work with V. Vasyunin.)

I will describe the Bellman-function approach to BMO defined on α-trees, which are structures that generalize dyadic lattices. Applications include the John–Nirenberg inequality and an inequality relating BMO_1 and BMO_2 norms on trees, with explicit constants. In particular, this gives the exact John–Nirenberg constant of the dyadic BMO on $\mathbb{R}^n$. The tools presented can be used on any function class that corresponds to a non-convex Bellman domain.

I. Videnskii. *An analog of Blaschke product for Hilbert space with Nevanlinna-Pick kernel.*

For a functional Hilbert space with Nevanlinna-Pick kernel, we define an abstract Blaschke condition, construct associate infinite product of multipliers and investigate its convergence in different topologies.

H. Woracek. *Perturbation of chains of de Branges spaces.*

We investigate the structure of the set of de Branges spaces of entire functions which are contained in a space $L^2(\mu)$. Thereby, we follow a perturbative approach. The main result is a growth dependent stability theorem: Assume that two measures μ_1 and μ_2 are close to each other in a sense quantified relative to a proximate order. Consider the sections of corresponding chains of de Branges spaces which consist of those spaces whose elements have finite (possibly zero) type w.r.t. the given proximate order. Then these sections can be related.

Cases of particular interest occur when a priori knowledge on the chains is available. For example, when it is known that all functions in de Branges spaces in the chain of either measure have order not exceeding a given constant < 1, or when for one of the measures it is known that the polynomials are dense in the corresponding L^2 space. Our results give rise to some stability assertions for the type of measure w.r.t. an order < 1 (where this notion is defined via the space in the corresponding de Branges chain).

D. Yakubovich. *Completeness of rank one perturbations of compact normal operators with lacunary spectrum.* (A joint work with A. Baranov.)

Let A be a compact normal operator on a Hilbert space H with certain lacunarity condition on the spectrum (which means, in particular, that its eigenvalues go to zero exponentially fast), and let L be its rank one perturbation. We show that either infinitely many moment equalities hold or the linear span of root vectors of L, corresponding to non-zero eigenvalues, is of finite codimension in H. In contrast to classical results, we do not assume that the perturbation is weak. We also give some examples, showing that the moments may fail to exist if the spectrum is not lacunary.

R. Zarouf. *Maximum of the resolvent over matrices with given spectrum.* (A joint work with O. Szehr.)

In numerical analysis it is often necessary to estimate the condition number $CN(T) = \|T\| \cdot \|T^{-1}\|$ and the norm of the resolvent

$\|(\zeta - T)^{-1}\|$ of a given $n \times n$ matrix T. We derive new spectral estimates for these quantities and compute explicit matrices that achieve our bounds. We recover the fact that the supremum of $CN(T)$ over all matrices with $\|T\| \leq 1$ and minimal absolute eigenvalue $r = \min\{\lambda \in \sigma(T) : \quad |\lambda| > 0\}$ is the Kronecker bound $\frac{1}{r^n}$. This result is subsequently generalized by computing for given ζ in the closed unit disc the supremum of $\|(\zeta - T)^{-1}\|$, where $\|T\| \leq 1$ and the spectrum $\sigma(T)$ of T is constrained to remain at a pseudo-hyperbolic distance of at least $r \in (0, 1]$ around ζ. We find that the supremum is attained by a triangular Toeplitz matrix. This provides a simple class of structured matrices on which condition numbers and resolvent norm bounds can be studied numerically. The occurring Toeplitz matrices are so-called model matrices, i.e. matrix representations of the compressed backward shift operator on the Hardy space H_2 to a finite-dimensional invariant subspace.

P. Zatitskiy. *On monotonic rearrangements of functions with small mean oscillation.* (A joint work with D. Stolyarov and V. Vasyunin.)

We will talk about monotone rearrangements in functional classes which can be expressed in special geometrical terms (the BMO space, Muckenhoupt classes, and Gehring classes are of such type). Moreover, we will talk about monotonic rearrangements from "dyadic-type" classes to "continuous" ones. In particular, we calculate the norm of monotonic rearrangement operation from the dyadic multidimensional BMO space to the one-dimensional BMO space.